Quiz 1

1. y1(x) and y2(x) are both solutions to a linear differential equation L1, what does that imply about

$$ay_1(x) + by_2(x)$$

This is also a solution of L1. (Superposition of solutions for linear differential equations).

2. Find the general solution of:

$$y'(x) + xy(x) = x$$

$$\lambda(x) = e^{\int x dx} = e^{\frac{x^2}{2}}$$

$$\frac{dy}{dx}e^{\frac{x^2}{2}} + y(x)xe^{\frac{x^2}{2}} = xe^{\frac{x^2}{2}} \Rightarrow e^{\frac{x^2}{2}}dy + \left[y(x)xe^{\frac{x^2}{2}} - xe^{\frac{x^2}{2}}\right]dx$$

$$y(x)e^{\frac{x^2}{2}} - e^{\frac{x^2}{2}} = C \Rightarrow y(x) = Ce^{-\frac{x^2}{2}} + 1$$

3. Find y(x) such that y(0) = 0, y(Pi/4) = 2 if:

$$y''(x) + 4y(x) = 0 \Rightarrow Ay''(x) + By'(x) + Cy(x) = 0$$

$$\frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \pm 2i$$

$$y(x) = Ae^{2ix} + Be^{-2ix}$$

$$y(0) = 0 \Rightarrow A + B = 0,\ y\left(\frac{\pi}{4}\right) = 2 \Rightarrow iA - iB = 2$$

$$A = \frac{1}{i},\ B = -\frac{1}{i} \Rightarrow y(x) = \frac{e^{2ix} - e^{-2ix}}{i} = 2Sin(2x)$$