Engr 5a

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Homework #7

Due next Wed

1. Express A^{-1}, A^2 and all powers of A as a linear combination of A and I, find e^{tA}:

$$\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} = A$$

2. Express A^{-1}, A^2 and all powers of A as a linear combination of A and I, find e^{tA}:

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = A$$

3. Express all powers of A as a linear combination of A^2, A and I, find e^{tA}:

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} = A$$

1. $A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$

$A^n = a_1 A + a_0 I$

$e^{tA} = ?$

$B = At = \begin{bmatrix} t & 0 \\ t & 2t \end{bmatrix}$

$e^B = a_1 B + a_0 I = \begin{bmatrix} a_1 t & 0 \\ a_1 t & 2a_1 t \end{bmatrix} + \begin{bmatrix} a_0 & 0 \\ 0 & a_0 \end{bmatrix}$

$e^B = \begin{bmatrix} a_1 t + a_0 & 0 \\ a_1 t & 2a_1 t + a_0 \end{bmatrix}$

find the eigenvalues of B

$\begin{vmatrix} t-\lambda & 0 \\ t & 2t-\lambda \end{vmatrix} = 0 = (t-\lambda)(2t-\lambda)$

$\lambda_1 = t \ , \ \lambda_2 = 2t$

$f(\lambda) = r(\lambda)$

$e^\lambda = a_1 \lambda + a_0$

$e^t = a_1 t + a_0$
$e^{2t} = a_1 2t + a_0$

solve for a_1 & a_0
$e^{2t} - e^t = a_1 t$

$a_1 = \frac{1}{t}(e^{2t} - e^t)$

$e^t = \frac{1}{\cancel{t}}(e^{2t} - e^t)\cancel{t} + a_0$
$2e^t - e^{2t} = a_0$

$\therefore \ e^{[A]t} = \begin{bmatrix} \cancel{e^{2t}} - e^t + 2e^t - \cancel{e^{2t}} & 0 \\ e^{2t} - e^t & 2e^{2t} - 2\cancel{e^t} + 2e^t - e^{2t} \end{bmatrix}$

$$\boxed{e^{[A]t} = \begin{bmatrix} e^t & 0 \\ e^{2t} - e^t & e^{2t} \end{bmatrix}}$$

	Homework # 7	Solutions

2. Express A^n as a linear combination of A & I find e^{tA}

$$A^n = a_1 A + a_0 I$$

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B = At = \begin{bmatrix} -t & 0 \\ 0 & t \end{bmatrix}$$

$$e^B = a_1 B + a_0 I = \begin{bmatrix} -a_1 t & 0 \\ 0 & a_1 t \end{bmatrix} + \begin{bmatrix} a_0 & 0 \\ 0 & a_0 \end{bmatrix} = \begin{bmatrix} -a_1 t + a_0 & 0 \\ 0 & a_1 t + a_0 \end{bmatrix}$$

find the eigenvalues of B

$$\begin{vmatrix} -t-\lambda & 0 \\ 0 & t-\lambda \end{vmatrix} = 0 = -(t+\lambda)(t-\lambda)$$

$$\lambda = t, -t$$

$$f(\lambda) = r(\lambda)$$

$$e^{\lambda} = a_1 \lambda + a_0$$

$$e^{t} = a_1 t + a_0$$
$$e^{-t} = -a_1 t + a_0$$

$$\tfrac{1}{2}(e^{t} + e^{-t}) = a_0$$

$$\tfrac{1}{2t}(e^{t} - e^{-t}) = a_1$$

$$\therefore e^{At} = \begin{bmatrix} \tfrac{1}{2}(-\cancel{e^{t}} + e^{-t}) + \tfrac{1}{2}(\cancel{e^{t}} + e^{-t}) & 0 \\ 0 & \tfrac{1}{2}(e^{t} \cancel{- e^{-t}} + e^{t} \cancel{+ e^{-t}}) \end{bmatrix}$$

$$\boxed{e^{At} = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{t} \end{bmatrix}}$$

3. $A^n = a_2 A^2 + a_1 A + a_0$

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B = At = \begin{bmatrix} 0 & t & t \\ 0 & t & t \\ 0 & 0 & 0 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 0 & t & t \\ 0 & t & t \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & t & t \\ 0 & t & t \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & t^2 & t^2 \\ 0 & t^2 & t^2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$e^B = a_2 B^2 + a_1 B + a_0 I = \begin{bmatrix} a_0 & a_2 t^2 + a_1 t & a_2 t^2 + a_1 t \\ 0 & a_2 t^2 + a_1 t + a_0 & a_2 t^2 + a_1 t \\ 0 & 0 & a_0 \end{bmatrix}$$

find the eigenvalues of B

$$\begin{vmatrix} -\lambda & t & t \\ 0 & t-\lambda & t \\ 0 & 0 & -\lambda \end{vmatrix} = 0 = \lambda^2 (t - \lambda)$$

$\lambda = 0, 0, t$

$e^{\lambda} = a_2 \lambda^2 + a_1 \lambda + a_0$

$e^{0} = a_2 0^2 + a_1 0 + a_0$
$e^{0} = 2a_2 0 + a_1$
$e^{t} = a_2 t^2 + a_1 t + a_0$

$a_1 = 1$
$a_0 = 1$
$a_2 = \frac{1}{t^2}(e^t - t - 1)$

$$\therefore e^{At} = \begin{bmatrix} 1 & (e^t - \cancel{t} - 1) + \cancel{t} & e^t - \cancel{t} - 1 + \cancel{t} \\ 0 & e^t - \cancel{t} - \cancel{1} + \cancel{t} + \cancel{1} & e^t - t - 1 + t \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{e^{At} = \begin{bmatrix} 1 & e^t - 1 & e^t - 1 \\ 0 & e^t & e^t - 1 \\ 0 & 0 & 1 \end{bmatrix}}$$

1. $\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} = A$

$\lambda = 1, 2$

$A^m = a_0 I + a_1 A$

$\lambda^m = a_0 + a_1 \lambda$

$1^m = a_0 + a_1$

$2^m = a_0 + 2a$

$2^m - 1 = a$

$a_0 = 1 - 2^m + 1 = 2 - 2^m$

$A^m = (2-2^m) I + (2^m - 1) A$

2. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = A$

$\lambda = 1, -1$

$A^m = a_0 I + a_1 A$

$\lambda^m = a_0 + a_1 \lambda$

$1 = a_0 + a_1$

$(-1)^m = a_0 - a_1$

$2a_0 = 1 + (-1)^m$

$a_0 = \frac{1+(-1)^m}{2}$

$a_1 = \frac{2-1-(-1)^m}{2} = \frac{1-(-1)^m}{2}$

$A^m = \left(\frac{1+(-1)^m}{2}\right) I + \left(\frac{1-(-1)^m}{2}\right) A$

3. $A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\lambda = 0, 0, 1$

$A^m = a_0 I + a_1 A + a_2 A^2$

$\lambda^m = a_0 + a_1 \lambda + a_2 \lambda^2$

$(0)^m = a_0 + a_1(0) + a_2(0)^2$

$m(0)^m = a_1 + 2a_2(0)$

$1 = a_0 + a_1 + a_2$

$a_0 = 0$

$a_1 = 0$

$a_2 = 1$

$A^m = a_2 A^2$

$\boxed{A^m = A^2}$