Homework #4

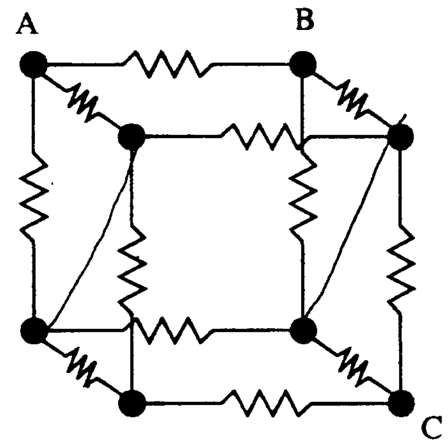
Due: Wed. at beginning of class.

Reading: Handout

Ref: Ma 5a Text, ECE 2a text

1. The following differential equation has one solution that is a polynomial in x, -- find the polynomial solution: $(2x - 3x^3)y'' + 4y' + 6xy = 0$

2. Consider the following resistor network:



Given that all resistors are 1Ω, find the resistances between nodes:

a) A to B

b) A to C

c) B to C

(Hint: use symmetry to find nodes that must be at a common potential.)

3. Suppose a 2A current source is placed so that a positive current flows into node A and out of node C, and that a 3A source is placed so that a positive current flows into node A and out of node B. What voltage will appear between points B and C? (Hint: use superposition!)

1. Find a polynominal solution to

$$(2x-3x^3)y''+4y'+6xy=0$$

Assume y is of the form $y=Ax^2+Bx+C$

$$y'=2Ax+B$$
$$y''=2A$$

plug back into ~~original~~ equation

$$2A(2x-3x^3)+4(2Ax+B)+6x(Ax^2+Bx+C)=0$$

$$x^3(-6A+6A)+x^2(6B\cancel{2})+x(4A+8A+6C)+4B=0$$

$$4B=0$$
$$12A+6C=0$$
$$6B=0$$

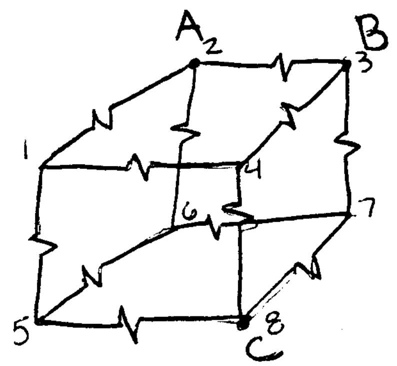
$$\therefore\ B=0$$

$$12A=-6C$$

$$C=-2A$$

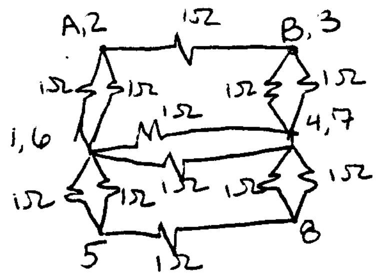
$$\boxed{\therefore y=Ax^2-2A}$$

2. Resistor Cube



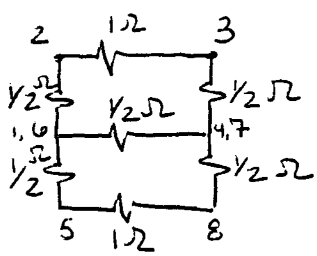
a) Find the resistance from A→B

from symmetry it is known that nodes 1&6 are at the same potential ~~as~~ And can be shorted together as can nodes 4 and 7

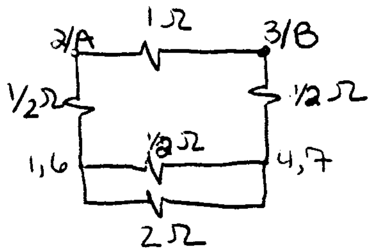


combine all the resistors in parallel

$1\Omega \parallel 1\Omega = \frac{1}{2}\Omega$



combine the bottom 3 in series

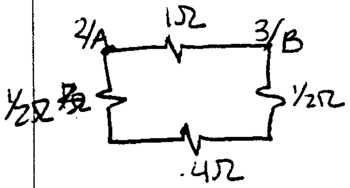


$\frac{1}{2}\Omega + 1\Omega + \frac{1}{2}\Omega = 2\Omega$

2. a) (cont)

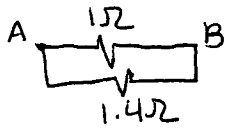
combine the resistors in parallel

$\frac{1}{2}\Omega // 2\Omega = .4\Omega$



combine the last series resistors

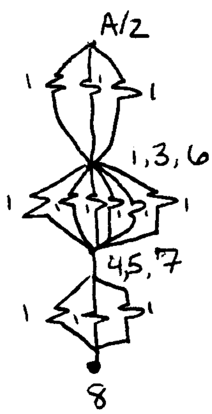
$\frac{1}{2}\Omega + \frac{1}{2} + .4\Omega = 1.4\Omega$



$R_{AB} = 1\Omega // 1.4\Omega = \boxed{\frac{7}{12}\Omega = .583\Omega = R_{AB}}$

b) from $A \to C$

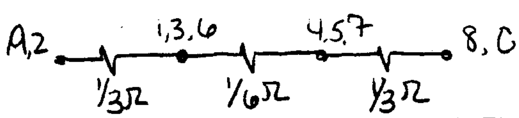
from symmetry it can be observed nodes 4, 5 + 7 are at the same potential as are 1, 3 + 6



combine the parallel resistors

$1\Omega // 1\Omega // 1\Omega = \frac{1}{3}\Omega$

$1\Omega // 1\Omega // 1\Omega // 1\Omega // 1\Omega // 1\Omega = \frac{1}{6}\Omega$

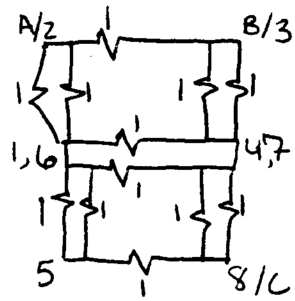


$R_{AC} = \frac{1}{3}\Omega + \frac{1}{6}\Omega + \frac{1}{3}\Omega = \boxed{\frac{5}{6} = R_{AC}}$

2 cont.

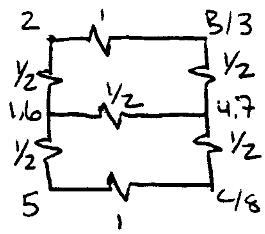
c) ~~R_{AB}~~ R_{BC}

from symmetry one can see 1 & 6 and 4 & 7 are at the same ~~potient~~ potential

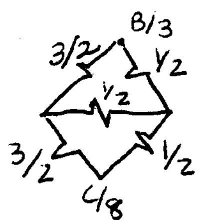


combine parallel resistors

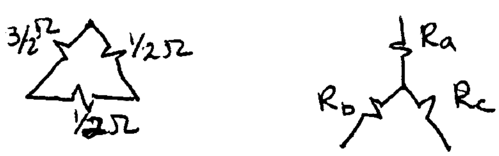
$1\Omega // 1\Omega = \frac{1}{2}\Omega$



combine series resistors



Using the Y-Δ transformation

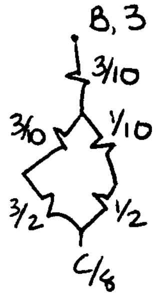


$$R_a = \frac{(3/2)(1/2)}{\frac{1}{2}+\frac{1}{2}+\frac{3}{2}} = \frac{3/4}{5/2} = \frac{3}{10}\Omega$$

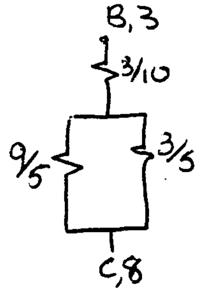
$$R_b = \frac{(3/2)(1/2)}{\frac{1}{2}+\frac{1}{2}+\frac{3}{2}} = \frac{3}{10}\Omega$$

$$R_c = \frac{(1/2)(1/2)}{3/2+\frac{1}{2}+\frac{1}{2}} = \frac{1/4}{5/2} = \frac{1}{10}\Omega$$

2 c) cont

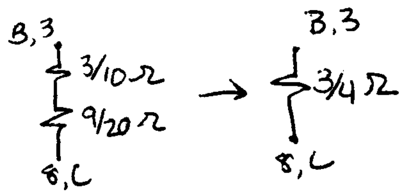


combine the series resistors



combine the parallel resistors

$\frac{9}{5}\Omega \,||\, \frac{3}{5}\Omega = \frac{9}{20}$



$\boxed{R_{BC} = \frac{3}{4}\,\Omega}$

#3

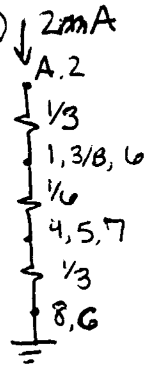
2A current into A & out of C
3A current into A & out of B

What is V_{BC}?

We will find V_{BC} for each current source then superpose them.

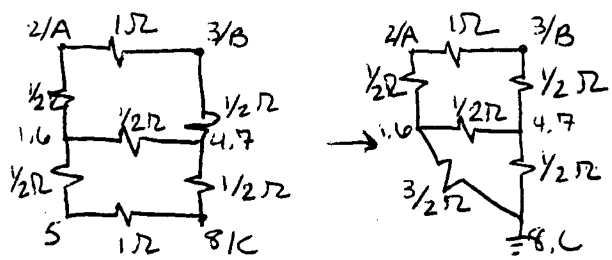
from (2)

R_{AC} =



$$V_{BC} = 2A\left(\tfrac{1}{3}+\tfrac{1}{6}\right) = 2A\left(\tfrac{1}{2}\right) = 1V$$

from (2)



Set up nodal equations

② $\cancel{3}-3A = \left(\frac{V_B - V_A}{1\Omega}\right) + \left(\frac{V_6 - V_A}{\frac{1}{2}\Omega}\right)$

③ $3A = \left(\frac{V_4 - V_B}{\frac{1}{2}\Omega}\right) + \left(\frac{V_A - V_B}{1\Omega}\right)$

⑧ $0 = \left(\frac{V_6 - \cancel{V_c}}{\frac{3}{2}\Omega}\right) + \left(\frac{V_4 - \cancel{V_c}}{\frac{1}{2}\Omega}\right)$

⑥ $0 = \left(\frac{V_A - V_6}{\frac{1}{2}\Omega}\right) + \left(\frac{V_4 - V_6}{\frac{1}{2}\Omega}\right) + \left(\frac{\cancel{V_8} - V_6}{\frac{3}{2}\Omega}\right)$

④ $0 = \left(\frac{V_B - V_4}{\frac{1}{2}\Omega}\right) + \left(\frac{V_6 - V_4}{\frac{1}{2}\Omega}\right) + \left(\frac{\cancel{V_8} - V_4}{\frac{1}{2}\Omega}\right)$

Set $V_8 = 0 = V_c$

$\frac{-V_6}{\frac{3}{2}\Omega} = \frac{V_4}{\frac{1}{2}\Omega}$ $\quad -V_6 = 3V_4$

$-3A = V_A(-1-2) + V_B + 2V_6$
$3A = V_A \quad -3V_B - \frac{2}{3}V_6$
$0 = 2V_A \quad 0 + V_6(-4 - \frac{2}{3} - \frac{2}{3})$

$V_B = -6/11\ V = -.545\ V$

$V_{BC} = 1V + -.545\ V = \boxed{.454V \cancel{\approx} = V_{BC}}$