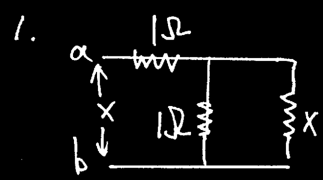
HW2 solutions

1.

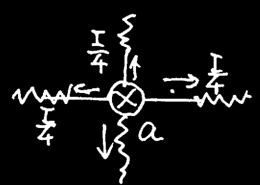


Assume the resistance is x

$\because \quad 1+\frac{x}{1+x}=x \Rightarrow x^2-x-1=0$

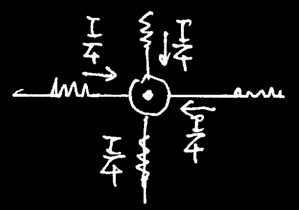
$\Rightarrow x=\frac{1\pm\sqrt{5}}{2}$, The reasonable value is $x=\frac{1+\sqrt{5}}{2}$

2. To a we assume a current I enter it

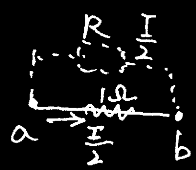


From the symmestry property of infinite network, we obtain each direction has $\frac{I}{4}$ current.

To b we assume a current I goes out of b.



Plus two infinite network, we get.



$\therefore \quad R_{ab}=R // 1\Omega$

$\because \quad I_R = I_{1\Omega} \Rightarrow R = 1\Omega$

$\therefore \quad R_{ab}=\frac{1}{2}\Omega$

3. Because of function $y'+P(x)y=Q(x)$, we have

$$y=e^{-\int P(x)dx}\cdot\left[\int Q(x)\cdot e^{\int P(x)dx}dx+C\right]$$

$$=\underbrace{e^{-\int P(x)dx}\cdot\int Q(x)\cdot e^{\int P(x)dx}dx}_{\text{special solution}}+\underbrace{C\cdot e^{-\int P(x)dx}}_{\text{general solution}} \qquad (1)$$

To function $xy'-2y=x^5$ ...... divide by x

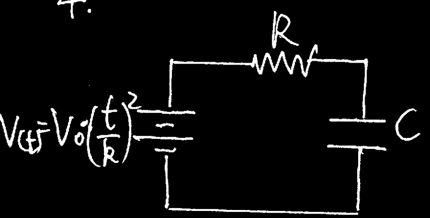
$$y'-2\frac{y}{x}=x^4$$

from (1) $\Rightarrow$

$$y=e^{-\int(\frac{-2}{x})dx}\cdot\int e^{\int(-\frac{2}{x})dx}\cdot x^4 dx + C\cdot e^{-\int(\frac{-2}{x})dx}$$

$$=\frac{1}{3}x^5+Cx^2$$

4.



From kirchoff's law, we get

$$V(t) = V_R(t) + V_C(t) \quad , \quad I_R(t) = I_C(t)$$

∴ To C : we have

$$I_C(t) = C \cdot \frac{dV_C(t)}{dt} = I_R(t) = \frac{1}{R}(V(t) - V_C(t))$$

$$\therefore \quad \frac{dV_C(t)}{dt} = \frac{1}{RC}(V(t) - V_C(t))$$

Because $V(t) = V_0(t/k)^2$ So we get equation :

$$y' + ay = bx^2 \quad (x = t,\ y = V_C(t),\ a = \frac{1}{RC},\ b = \frac{V_0}{RCk^2})$$

$$\therefore \quad y = e^{-\int a\,dx} \int bx^2 \cdot e^{\int a\,dx} dx + C_1 \cdot e^{-\int a\,dx}$$

$$= e^{-ax} \cdot \int bx^2 e^{ax} dx + C_1 e^{-ax}$$

$$= C_1 e^{-ax} + \frac{bx^2}{a} - \frac{2b}{a^2} x + \frac{2b}{a^3}$$

$$\therefore \quad \cancel{y} \quad V_C(t) = \frac{V_0}{k^2} t^2 - \frac{2RC}{k^2} V_0 t + \frac{2R^2C^2V_0}{k^2} + C_1 e^{-\frac{t}{RC}}$$

Becaue when $t=0$ $V_C(t) = 0 \Longrightarrow C_1 = -\frac{2R^2C^2V_0}{k^2}$

$$\therefore \quad V_C(t) = \frac{2RCV_0}{k^2}\left(\frac{1}{2RC} t^2 - t + RCV_0 - RC \cdot e^{-\frac{t}{RC}}\right)$$

5. $y'' + P(x) y' + Q(x) y = R(x)$

Let $y = uv$ we obtain $y' = u'v + v'u$; $y'' = u''v + 2v'u' + v''u$

$$\therefore \quad \underline{u''v} + 2u'v' + v''u + \underline{P(x)u'v} + P(x)v'u + \underline{Q(x)uv} = R(x)$$

$$\because \quad v(u'' + P(x)u' + Q(x)u) = 0$$

we get $v''u + 2v'u' + P(x)v'u = R(x)$

let $z = v' \Longrightarrow z'u + (2u' + P(x)u) \cdot z = R(x)$