

1.(a)  $e^{i\left(2\pi n + \frac{3\pi}{2}\right)}$  n: interger

(b)  $\sqrt{29}e^{i\left(2\pi n + \text{atan}\left(\frac{5}{2}\right)\right)}$

(c)  $\sqrt[n]{e^{i\left(2\pi n + \frac{\pi}{2}\right)}} = e^{i\left(\pi n + \frac{\pi}{4}\right)}$

(d)  $\cos\left(\frac{1}{2}\right) - i\sin\left(\frac{1}{2}\right) = e^{i\left(2\pi n - \frac{1}{2}\right)}$

2. (a)  $-3(\cos(2) + i\sin(2))$

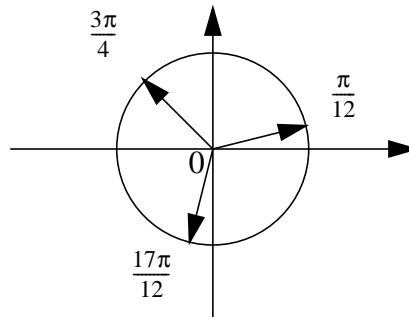
(b)  $2(\cos(\pi) + i\sin(\pi)) = -2$

(c)  $e^{(a+ib)} = e^a e^{ib} = e^a(\cos(b) + i\sin(b))$

(d)  $\frac{i}{a-i} = \frac{i(a+i)}{(a-i)(a+i)} = \frac{1}{a^2+1}(-1+ia)$

3. (a) The roots are  $\sqrt[6]{2}\left(e^{i\left(2\pi n + \frac{\pi}{4}\right)}\right)^{\frac{1}{3}} = \sqrt[6]{2}e^{i\left(\frac{2\pi n}{3} + \frac{\pi}{12}\right)}$  n=0,1,2

The radius of circle is  $\sqrt[6]{2}$

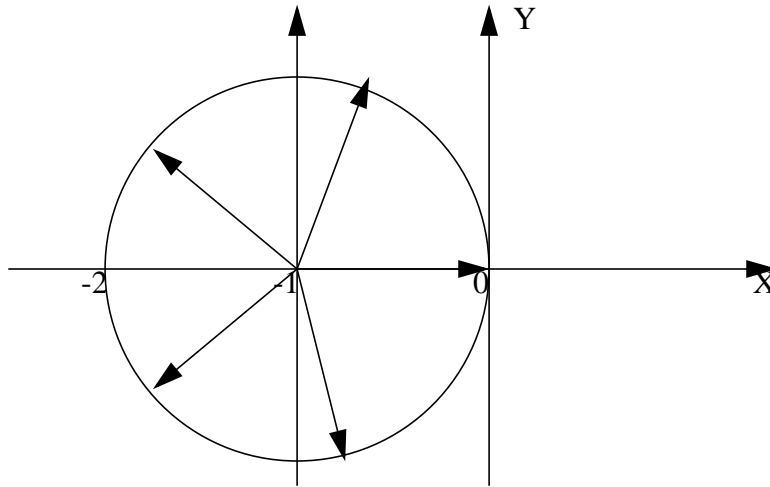


(b) Because  $e^{i2\pi n} = 1$

$$(Z+1)^5 = e^{i2\pi n} \rightarrow Z+1 = e^{i\frac{(2\pi n)}{5}} \rightarrow Z = e^{i\frac{(2\pi n)}{5}} - 1$$

n=0,1,2,3,4

$$Z = \left( \cos\left(\frac{2\pi n}{5}\right) - 1 \right) + i \sin\left(\frac{2\pi n}{5}\right) \quad n=0,1,2,3,4$$



4.

$$i^i = e^{i \ln i} = e^{i \ln \left( e^{i \left( 2\pi n + \frac{\pi}{2} \right)} \right)} = e^{-\left( 2\pi n + \frac{\pi}{2} \right)}$$

5.

$$\cos(Z) = 1 - \frac{Z^2}{2!} + \frac{Z^4}{4!} - \frac{Z^6}{6!} + \dots$$

$$\cos(i) = 1 + \frac{1}{2!} + \frac{1}{4!} + \dots + \frac{1}{(2n)!} + \dots$$

Because  $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$

$$e^1 = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!} + \dots$$

$$e^{-1} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \dots + \frac{(-1)^n}{n!} + \dots$$

So  $\cos(i) = \frac{e^{-1} + e}{2}$