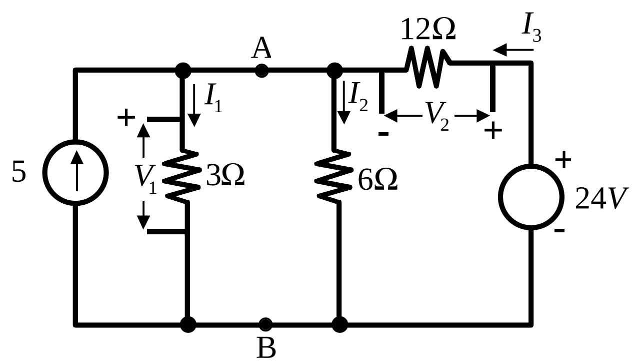
Kirchoff's Laws Direct:

KCL, KVL, Ohm's Law $\Rightarrow$ $V = IR \Rightarrow VG = I$
$G = R^{-1}$

<u>Ohm's Law:</u> $V_1 = 3 \cdot I_1 = 6 \cdot I_2$
$V_2 = 12 \cdot I_3$

(always get 1 equation/Resistor)



KCL:

A: $-5 + I_1 + I_2 - I_3 = 0$
B: $5 - I_1 - I_2 + I_3 = 0$

eq. are dependent

(in general, get n-1 indep. for nodes)

KVL: $-V_1 + 24 - V_2 = 0$ write 1 loop equation for each loop with a voltage not in the current set of equations.

$\Rightarrow$ Eliminate either V_1 or I using Ohm's Law

eq: A: $-5 + \frac{V_1}{3} + \frac{V_1}{6} - \frac{V_2}{12} = 0$

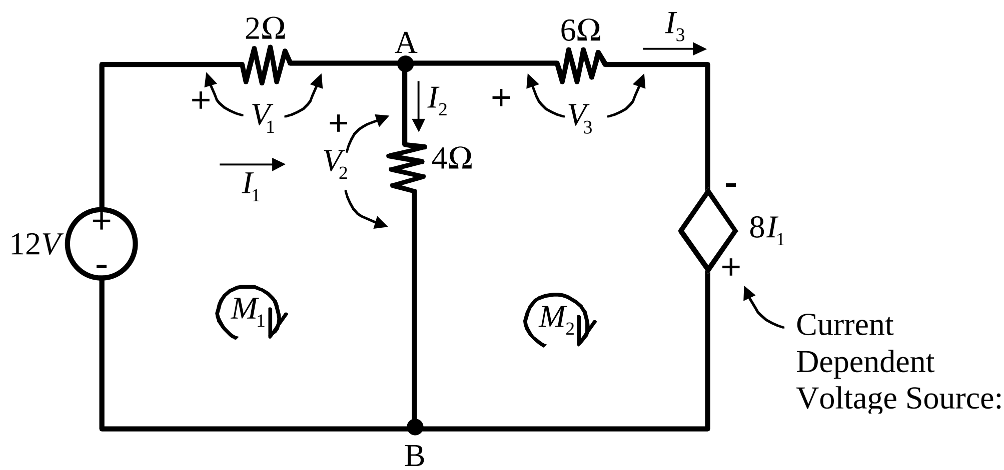
$$\Rightarrow \begin{bmatrix} \left(\frac{1}{3}+\frac{1}{6}\right) & \frac{-1}{12} \\ -1 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 24 \end{bmatrix}$$

KVL: $-V_1 + 24 - V_2 = 0$

Cramer's Rule: $[A] \cdot \vec{X} = \vec{B}$

$$X_i = \frac{Det\left[A^{0..i-1} | B | A^{i+1..N}\right]}{Det[A]}$$

We can always write in terms of only V, or I variables using ohm's law:



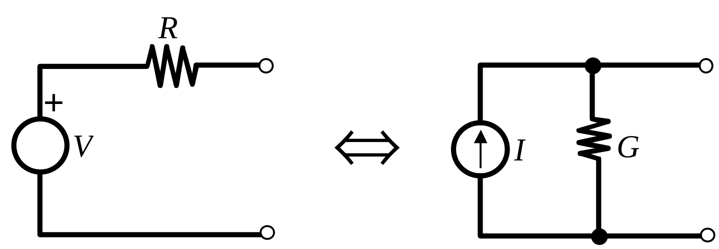
KCL: $\underline{A}$: $-I_1 + I_2 + I_3 = 0$

KVL: M_1: $-12 + 2I_1 + 4I_2 = 0$

M_2: $-4I_2 + 6I_3 - 8I_1 = 0$

$$\begin{bmatrix} -1 & 1 & 1 \\ 2 & 4 & 0 \\ -8 & -4 & 6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 0 \end{bmatrix}$$

Note:

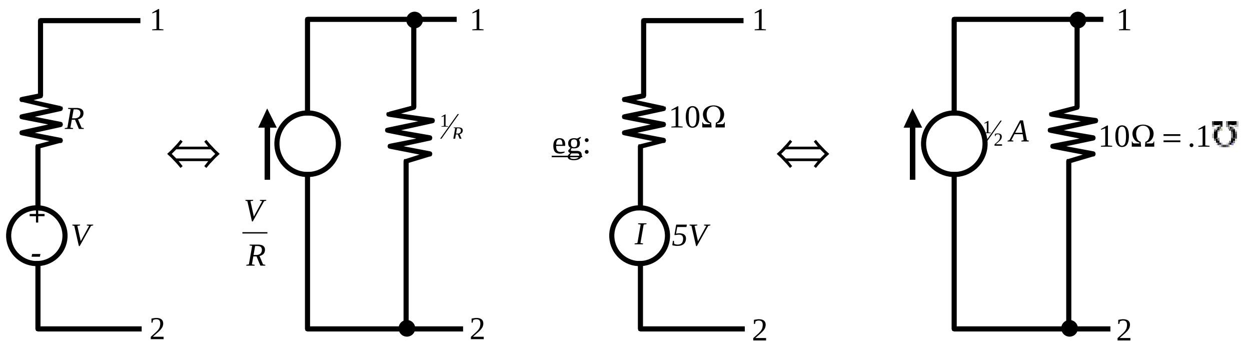


if $V = IR$

$R = G^{-1}$

Rules for <u>nodes</u>:

1) Convert all <u>voltage</u> to <u>current sources</u>



1) Determine a reference node and identify unknown relative voltages

2) Use KCL at each unknown node:

at node A: $G_{AA}V_A - G_{AB}V_B - .... - G_{AN}V_N = I_A$

B: $-G_{AB}V_A + G_{BB}V_B - .... - G_{BN}V_N = I_B$

N: $-G_{AN}V_A + G_{BN}V_B - .... - G_{NN}V_N = I_N$

where: $G_{ii} = \Sigma$ all conductances connected to node i

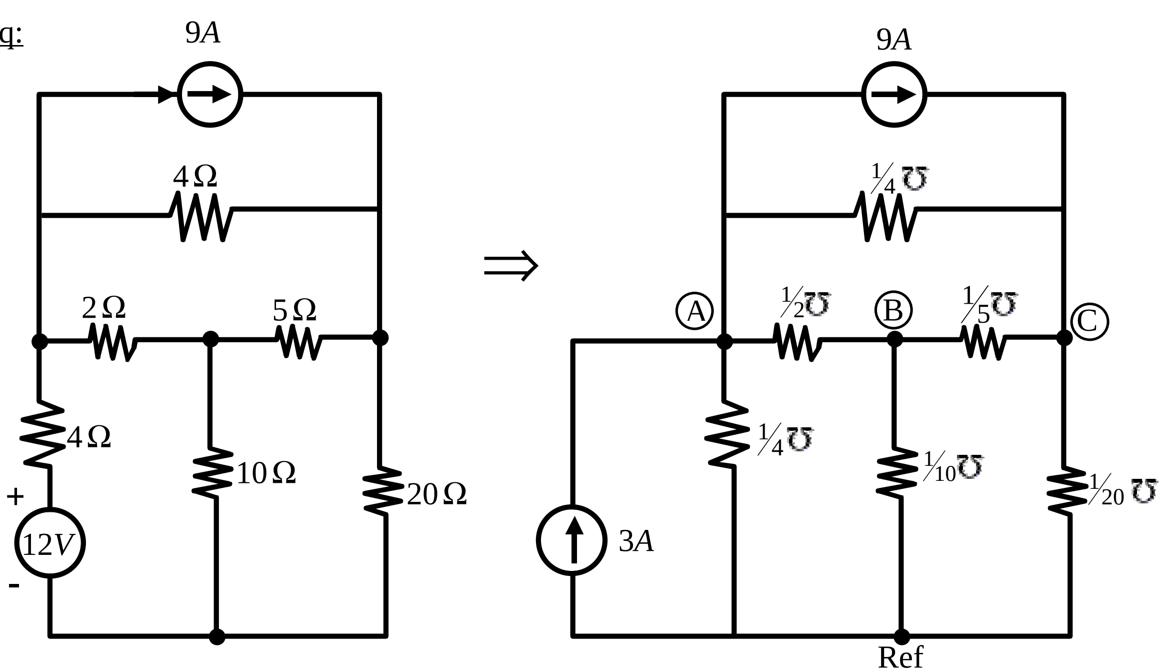
$G_{ji} = \Sigma$ all conductances between node i and node j

$I_i = \Sigma$ all current sources connected to node i

$\Rightarrow$ Define N equations in N unknowns

all other voltages and currents by Ohm's Law from V and S.

<u>Eq:</u>



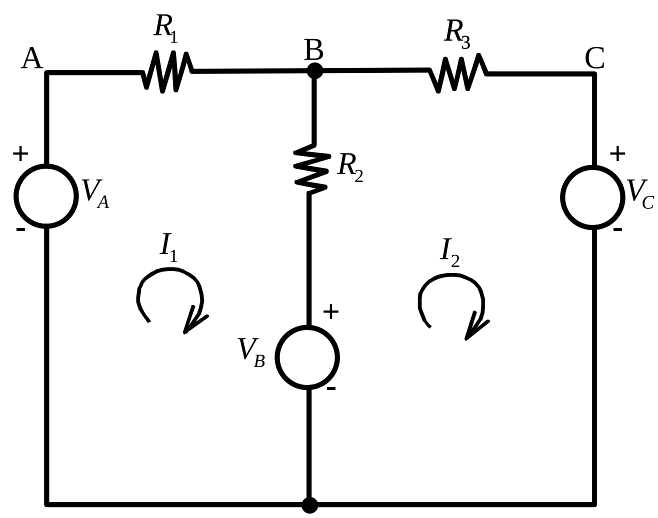
A: $(0.25+.5+.25)V_A - 0.5V_B - 0.25V_C = 3-9\ (A)$
B: $-0.5V_A + (0.5+0.1+0.2)V_B - .02V_C = 0$
C: $-.25V_A - 0.2V_B + (0.2+0.25+0.05)V_C = 9$

$$\Rightarrow \begin{bmatrix} 1.0 & -0.5 & -2.5 \\ -.5 & .8 & -.2 \\ -.25 & -.2 & .5 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ 9 \end{bmatrix}$$

$$\Rightarrow V_A = 4V \quad V_B = 8.33V \quad V_C = 23.3V$$

One can also solve for N independent currents in N meshes

Consider:



We know that KVL $\Rightarrow$
voltages around a loop = 0.

Current I_1 flows in a simple mesh.

I_1: $I_1 \cdot R_1 + I_1 \cdot R_2 - I_2 \cdot R_2 + V_B - V_A = 0$
I_2: $I_2 \cdot R_3 + I_2 \cdot R_2 - I_1 \cdot R_2 + V_C - V_B = 0$

We can as usual re-arrange to a form that can be written by inspection:

1: $(R_1+R_2)I_1 - R_2 I_2 = V_A - V_B$
2: $-R_2 I_1 + (R_2+R_3)I_2 = V_B - V_C$

$$\Rightarrow \begin{bmatrix} R_1+R_2 & -R_2 \\ -R_2 & R_2+R_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_A - V_B \\ V_B - V_C \end{bmatrix}$$

* Note that KCL is always solved implicitly since at each node we have a sum of currents in meshes: each mesh enters and leaves with the same current.

In general:

1) Connect each current source with parallel res. to voltage source with series *R*.

2) Select a current variable and mesh for each simple loop (usually we traverse each loop in <u>same direction</u>, ie, clockwise.

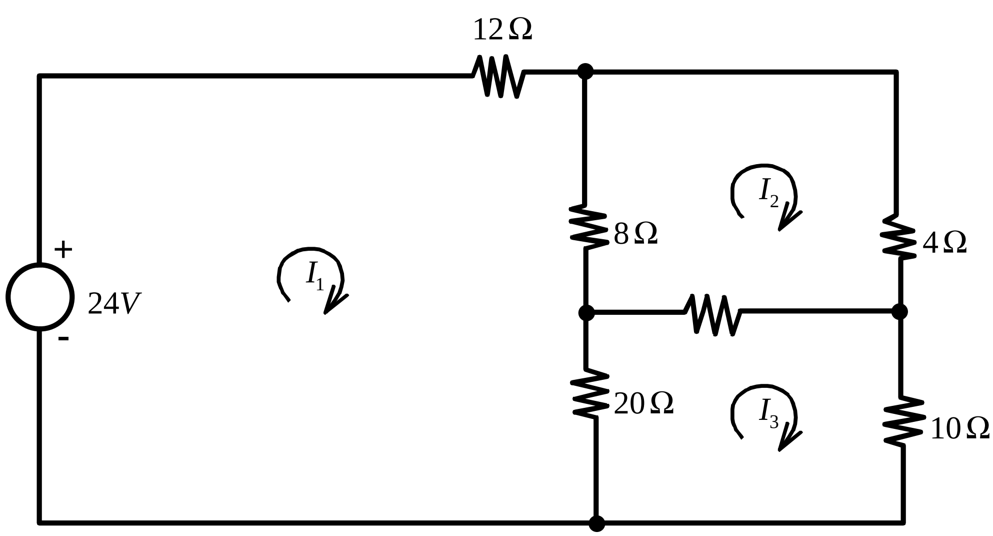
3) Use KVL for each loop in terms of the mesh <u>current variable</u>.

   iff no <u>dependent</u> sources:

   1: $R_{11}I_1 - R_{12}I_2 - ... - R_{1N}I_N = V_1$
   2: $-R_{12}I_1 - R_{22}I_2 - ... - R_{2N}I_N = V_2$
   N: $-R_{1N}I_1 - R_{2N}I_2 - ... - R_{NN}I_N = V_N$

   $R_{ii} =$ sum of all resistance in mesh *I*
   $R_{ij} =$ sum of all common resistance to meshes *I,J*
   $V_I =$ sum of voltage rises in mesh *I*, in direction of current I_1

<u>Eq</u>: A Wheatstone Bridge



$$\Rightarrow \quad (12+8+20)I_1 - 8I_2 - 20I_3 = 24$$
$$-8I_1 + (8+4+6)I_2 - 6I_3 = 0$$
$$-20I_1 - 6I_2 + (20+6+10)I_3 = 0$$

$$\begin{bmatrix} 40 & -8 & -20 \\ -8 & +18 & -6 \\ -20 & -6 & +36 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ 0 \end{bmatrix}$$

How to solve this system?

$\Rightarrow$ Gaussian Elimination to Triangular form:

$$\begin{matrix}1:\\2:\\3:\end{matrix}\begin{bmatrix}20 & -4 & -10\\ -4 & 9 & -3\\ -10 & -3 & 18\end{bmatrix}=\begin{bmatrix}12\\0\\0\end{bmatrix}$$

(divide both sides by 2)

$$3\cdot \tfrac{5}{9}=\begin{bmatrix}\frac{-80}{9} & \frac{-5}{3} & +10\end{bmatrix}\ [0]$$

$$\begin{matrix}1'\\2\\3\end{matrix}\begin{bmatrix}\frac{130}{9} & \frac{-17}{3} & 0\\ -4 & 9 & -3\\ -10 & -3 & 18\end{bmatrix}=\begin{bmatrix}12\\0\\0\end{bmatrix}$$

$$3\cdot \tfrac{1}{6}=\begin{bmatrix}\frac{-10}{6} & \frac{-1}{2} & 3\end{bmatrix}=[0]$$

$$\begin{matrix}1'\\2'\\3\end{matrix}\begin{bmatrix}\frac{130}{9} & \frac{-17}{3} & 0\\ \frac{-17}{5} & \frac{17}{2} & 0\\ -10 & -3 & 18\end{bmatrix}=\begin{bmatrix}12\\0\\0\end{bmatrix}$$

$$2'\cdot\frac{2}{17}\cdot\frac{17}{3}=2'\cdot\frac{2}{3}=\begin{bmatrix}\frac{-34}{9} & \frac{17}{3} & 0\end{bmatrix}$$

$$\begin{matrix}1'\\2''\\3\end{matrix}\begin{bmatrix}\frac{96}{9} & 0 & 0\\ \frac{-17}{3} & \frac{17}{2} & 0\\ -10 & -3 & 18\end{bmatrix}=\begin{bmatrix}12\\0\\0\end{bmatrix}$$

$$\Rightarrow \qquad \frac{96}{9}\cdot I_1=12V \quad I_1=\tfrac{5}{4}A$$

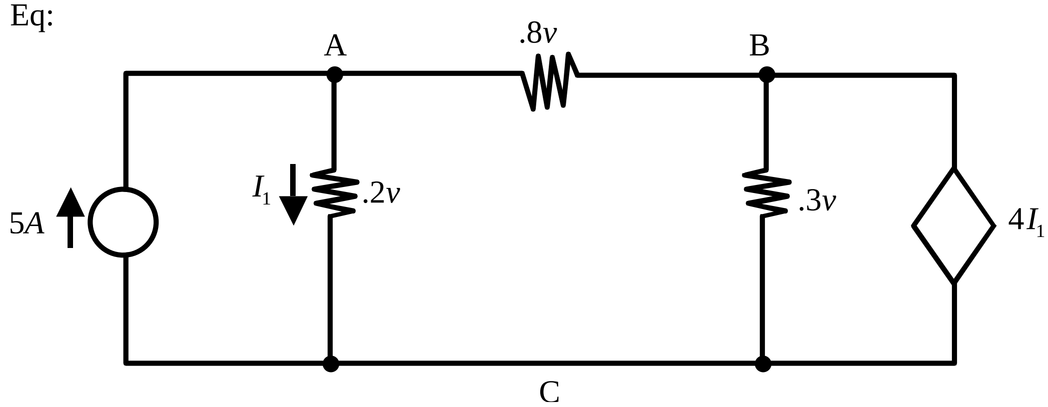
Then $\quad 2'=\frac{+17}{3}\cdot\frac{5}{4}=\frac{17}{2}\cdot I_2 \Rightarrow I_2=\tfrac{3}{4}A$

$3'\Rightarrow \quad +10\cdot\tfrac{5}{4}+3\cdot\tfrac{3}{4}=18\cdot I_3 \quad \Rightarrow \mathrm{I}_3=\tfrac{3}{4}A$

Dependent Voltage & Current Sources:

We model the activity of many active components by use of a "programmable" voltage or current source whose strength is a function of the voltage or currents elsewhere in the circuit:

Eq:



if we choose B as the reference node $\Rightarrow$

A: $(.8+.2)V_A - .2V_C = 5$

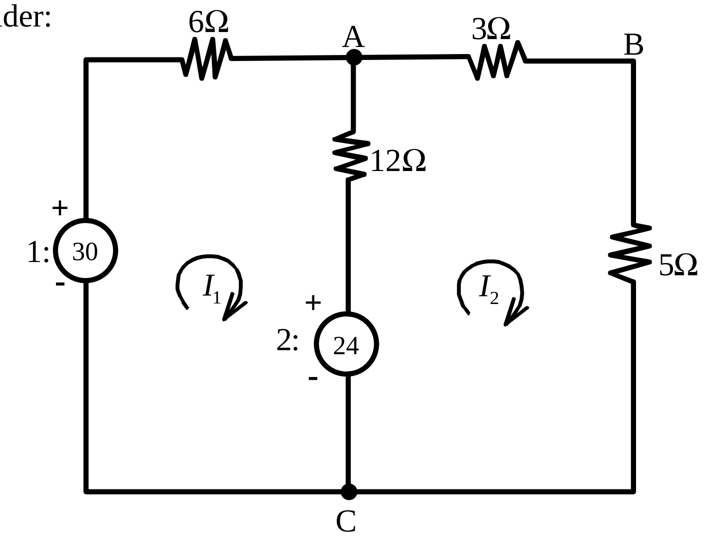
C: $-(.2)V_A + (.2+.3)V_C = -5 + 4I_1$

now $I = 0.2 \cdot (V_A - V_C)$

$$\Rightarrow \quad \begin{bmatrix} 1 & -.2 \\ -1 & 1.3 \end{bmatrix} \begin{bmatrix} V_A \\ V_C \end{bmatrix} = \begin{bmatrix} s \\ -s \end{bmatrix}$$

Superposition for Circuits

Consider:

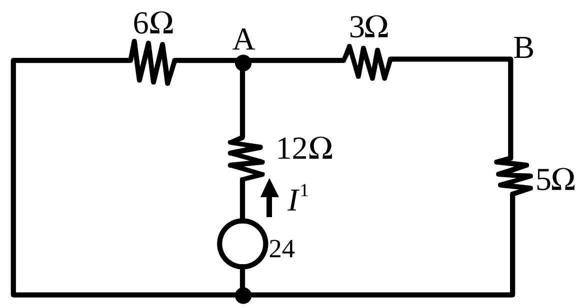


We could solve this by either node or mesh analysis, but there may be a simpler approach:

If we suppress source #1 (i.e. make a short circuit) we can find I_1', I_2'. Similarly, I_1, I_2 could be written without source #2.

Total currents and voltages superpose $\Rightarrow$ suppress one at a time and then superpose the results:

Suppress 1:

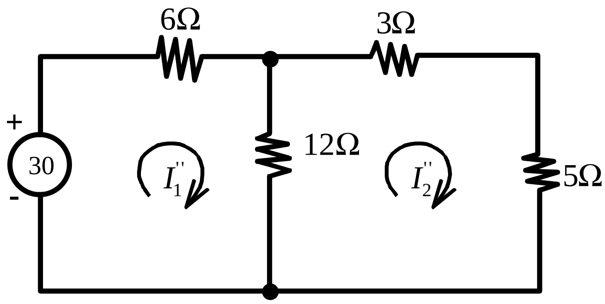


$$\Rightarrow\ 24V = \left(12 + 6\|(3+5)\right)\cdot I'$$

So $I' = \frac{7}{108}\cdot 24 = \frac{14}{9}A \Rightarrow V_A' = \frac{56}{3} + 24$ volt

$$I_1' = \frac{-128}{18} \qquad I_2' = \frac{128}{18}$$

Now: Suppress I_2, run I_1

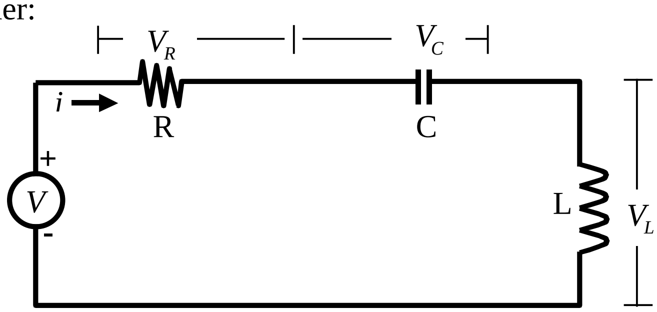


$$30V = \left(6 + 12 \| (8)\right) \cdot I_1''$$

Exponential Excitation of circuits: Admittance & Impedance

Idea: Exponential function is easy to analyze $\Rightarrow$
simple to add/multiply/integrates, etc.
Also is a common case for circuit excitation

Consider:



where $V = Ae^{st}$ for <u>complex</u> s, A, real t.

We know all <u>currents</u> are same in circuit, $V_R + V_C + V_L = V$ <u>KVL</u>

$$V_R = i(t)\cdot R \qquad V_C = \frac{1}{C}\cdot\int i(t)dt \qquad V_L = L\frac{di(t)}{dt}$$

or $$i_R(t) = \frac{V_R(t)}{R} \qquad i_L(t) = C\frac{dV_C(t)}{dt} \qquad i_L(t) = \frac{1}{L}\int V_C(t)dt$$

for our circuit $V(t) = Ae^{st} \Rightarrow V_R(t) = A'e^{st},\ V_C(t) = A''e^{st}\ldots$

where: $Ae^{st} = (A' + A'' + A''')e^{st}$ <u>KVL</u>

we have $i_R(t) = \frac{A'}{R}e^{st}$ $i_C(t) = C\cdot A''\cdot s\, e^{st}$ $i_L = \frac{A'''e^{st}}{sL}$

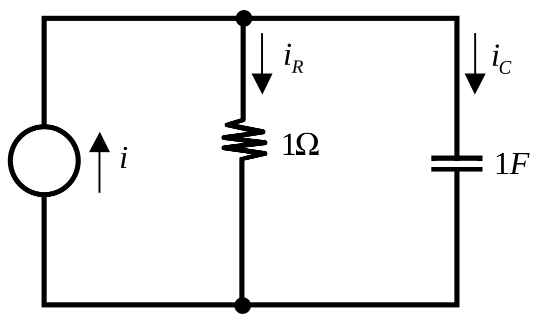
Note: for this kind of excitation, $i_L(t) = C \cdot A'' \cdot s\, e^{st}$
$= sC \cdot V_C(t)$

$$i_L(t) = \frac{1}{s^L} V_L(t)$$

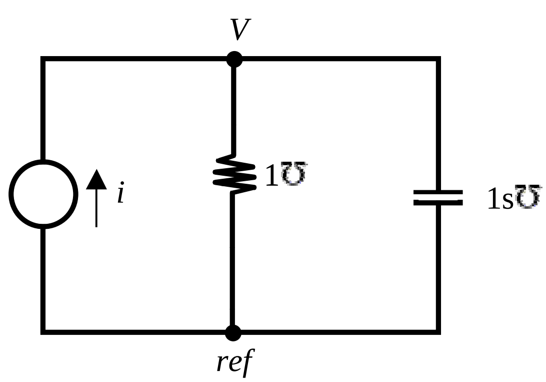
$$i_R(t) = \frac{1}{R} V_R(t)$$

So: sC has same units/behavior as $\frac{1}{R}$: conductance.

Eg. 2:



Parallel $\Rightarrow$ admittances $\Rightarrow$



$$1 = \frac{V}{1} + sv$$

$$\Rightarrow V = \frac{I}{1+s}$$

for $\quad i(t) = 10e^{-2t} \qquad s = -2, A = 10$

$$V = \frac{10}{1-2} = -10e^{st} = -10e^{-2t} \text{ volts}$$

Sinewave (Sinusodial) excitation:

We know: $e^{io} = \cos o + i \sin o$

$$\Rightarrow \quad \cos o = \frac{e^{io} + e^{-io}}{2} \qquad \sin o = \frac{e^{io} - e^{-io}}{2i}$$

We wish to study circuits with excitation: $v(t) = V\cos(wt + \phi)$

$$= \frac{V}{2}\left(e^{iwt+i\varphi} + e^{-iwt-i\varphi}\right) = v_i e^{iwt} + v_2 e^{-iwt}$$

$$v_1 = \frac{V}{2}e^{i\varphi} \qquad v_2 = \frac{V}{2}e^{-i\varphi}$$

note V is complex.

We define:

		R	C	L
	Admittance	$1/R$	sC	$1/sL$
	Impedance	R	$1/sC$	sL
for $s = iw$	(sinusoid case)	$1/R$	iwC	$1/iwL$
		R	$1/iwC$	iwL

Admittances compose like conductances, Impedances compose like resistances.

So for our circuit: $v(t) = R \cdot i(t) + L \cdot \frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt$

for $i(t) = Ie^{st}$

$$\Rightarrow v(t) = \left(R \cdot I + L \cdot I \cdot s + \frac{I}{sC}\right)e^{st}$$

$$\text{or } I = \frac{V}{R + Ls + 1/sC} = \frac{V}{z(s)}$$

we write <u>impedances as</u> $Z(s)$, admittance as *Y(s)*

so $i(t) = Ie^{st} = \frac{V}{R + Ls + 1/sC} \cdot e^{st}$

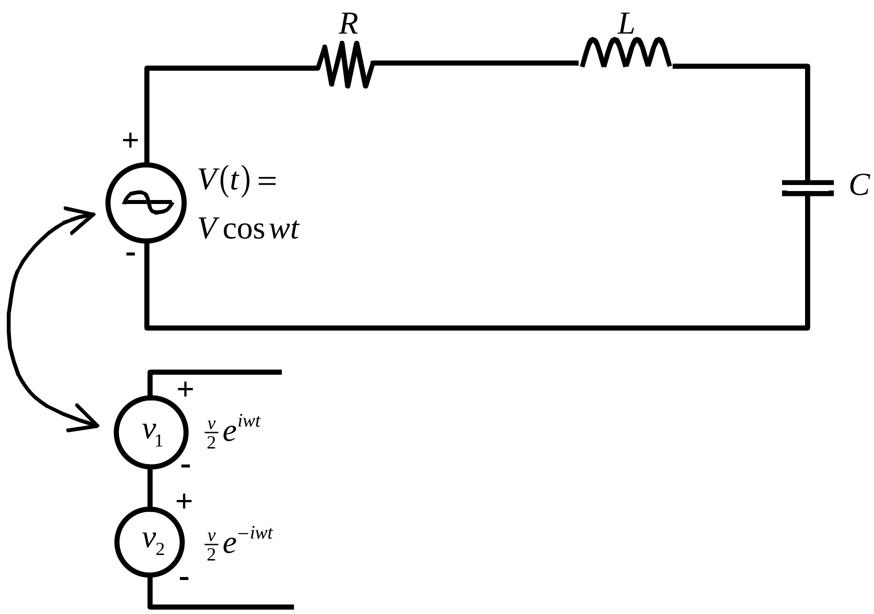
<u>eq</u>: if $V = 10 \cdot e^{-2t}$ $(10V\ @t = 0 \rightarrow \text{decreasing})$

$R = 1\Omega$ $L = \frac{1}{2}H$ $C = \frac{1}{2}F$

$$i(t) = \frac{10}{1 + s/2 + 2/s}e^{st} = -10e^{-2t} \text{ (Amps)}$$

Basic Trick: Extend the circuit techniques for node and mesh analysis to also handle Impedence and Admittances

$\Rightarrow$ generalize circuits which can be analyzed.



note: we can apply <u>superposition</u> to solve this:

suppress v_1 or v_2 and solve for other.

$$I_1 = \frac{V/2}{R + iwL + 1/iwC} \qquad I_2 = \frac{V/2}{R - iwL + 1/{-iwC}}$$

if we write I_1, I_2 in exponential complex form:

$$I_1 = I_1 e^{i\sigma_1} \qquad I_2 = I_2 e^{i\sigma_2}$$

we get: $I_1 = I_2 = \dfrac{V/2}{\sqrt{R^2 + (wL - 1/wC)^2}}$ $\qquad o_1 = -\tan^{-1}\left(\dfrac{wL - 1/wC}{R}\right) = -\sigma_2$

$$i_1(t) = I_1 e^{i\sigma_1} e^{iwt} \ldots.$$

$$i_1(t) = i_1 + i_2 = I_1\left(e^{i(wt+\sigma_1)} + e^{-i(wt-\sigma_1)}\right)$$

$$= \frac{V}{\sqrt{R^2 + (wL - 1/wC)^2}} \cdot \cos(wt + \sigma_1)$$

I, V are called <u>Phasors</u>