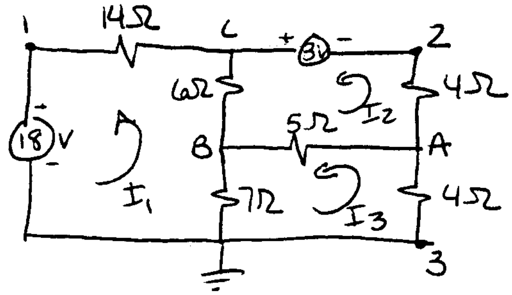
Homework #5 Solutions

1. find ~~VA~~ V_A, V_B, V_C



Constraints

$V_1 = 18V$
$V_3 = 0V$

Mesh

$$0 = -7\Omega(I_1 - I_3) - 6\Omega(I_1 - I_2) - 14\Omega I_1 - 18$$

$$0 = -4\Omega I_3 - 5\Omega(I_3 - I_2) - 7\Omega(I_3 - I_1)$$

$$0 = 4\Omega I_2 + 3V - 6\Omega(I_2 - I_1) - 5\Omega(I_2 - I_3)$$

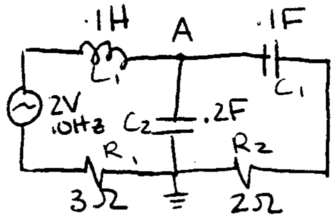
$$-27 I_1 + 6 I_2 + 7 I_3 = 18$$
$$7 I_1 + 5 I_2 - 16 I_3 = 0$$
$$6 I_1 - 15 I_2 + 5 I_3 = -3$$

$I_1 = -.85\ A$
$I_2 = -.297\ A$
$I_3 = -.466\ A$

$V_c = 18V + 14\Omega(.85) = \cancel{21.4V} = V_c = 29.\cancel{8}9V$
$V_A = 4\Omega(.466) = 1.864V = V_A$
$V_B = 7\Omega(.85 - .466) = 2.7V = V_B$

2. Find the voltage and phase at V_A

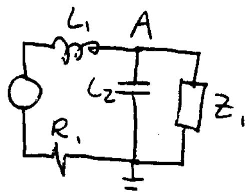
Voltage due to 2V 10Hz source



$\omega = 10Hz$

Combine C_1 & R_2 in ~~parallel~~ series

$$Z_{C_1R_2} = 2\Omega + \frac{1}{j\omega(.1)} = Z_1$$



Combine L_2 & Z_1 in parallel

$$\frac{1}{Z_2} = j\omega(.2) + \frac{1}{2 + \frac{1}{j\omega(.1)}}$$

$$Z_2 = \cancel{.497 + 12.6i}\ .00312 - .0792j$$

$$V_A = Im\left[2e^{i2\pi 10t}\left(\frac{Z_2}{Z_2 + R_1 + L_1}\right)\right]$$

$$= Im\left[2e^{i20\pi t}\left(\frac{.00312 - .0792j}{.00312 - .0792j + 3 + (20\pi)(.1)j}\right)\right]$$

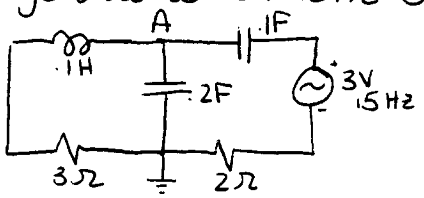
$$= Im\left[2e^{i20\pi t}(-.01015 - .00541j)\right]$$

$$= Im\left[2e^{i20\pi t}(.0115e^{-2.65j})\right]$$

$$V_A = .023 \sin(20\pi t - 2.65)$$

2 (cont)

Voltage due to 3V 15Hz source



Combine .1H in series with 3Ω

$$Z_1 = (.1)(2\pi 15)j + 3 = 9.42j + 3$$

Combine Z_1 in parallel w/ .2F

$$\frac{1}{Z_2} = j(30\pi)(.2) + \frac{1}{3 + 9.42j}$$

$$Z_2 = \cancel{1/.3}\ 87.3\times10^{-6} - .0533j$$

$$V_A = \overset{\text{Im}}{\cancel{\text{Re}}}\left[3e^{j30\pi t}\left(\frac{Z_2}{Z_2 + 2 + \frac{1}{.1(30\pi)j}}\right)\right]$$

$$= \overset{\text{Im}}{\cancel{\text{Re}}}\left[3e^{j30\pi t}\left(.00216 - .0265j\right)\right]$$

$$= \overset{\text{Im}}{\cancel{\text{Re}}}\left[3e^{j30\pi t} \cdot .0266e^{-1.49j}\right]$$

$$V_A = .0797 \underset{\sin}{\overset{\cos}{\cancel{\cos}}}(30\pi t - 1.49)$$

Superposition

$$V_{A_{tot}} = .0797 \overset{\sin}{\cancel{\cos}}(30\pi t - 1.49) + .023 \overset{\sin}{\cancel{\cos}}(20\pi t - 2.65)$$

3. find a general solution to

$$\begin{bmatrix} 2 & -3 \\ 4 & +6 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_1' \\ y_2' \end{bmatrix}$$

a. find the eigenvalues

$$\begin{vmatrix} 2-\lambda & -3 \\ 4 & 6-\lambda \end{vmatrix} = 0 = (2-\lambda)(6-\lambda)+12$$

$$= 12 - 8\lambda + \lambda^2 + 12$$

$$= 24 - 8\lambda + \lambda^2$$

eigenvalues are $\lambda = 4 + 2\sqrt{2}\,i;\ 4 - 2\sqrt{2}\,i$

find eigenvectors

$\lambda = 4 + 2\sqrt{2}\,i$

$$\begin{bmatrix} 2-(4+2\sqrt{2}\,i) & -3 \\ 4 & 6-(4+2\sqrt{2}\,i) \end{bmatrix} \rightarrow \begin{bmatrix} -2-2\sqrt{2}\,i & -3 \\ 4 & 2-2\sqrt{2}\,i \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & \frac{1}{2} - \frac{\sqrt{2}}{2}i \\ 0 & 0 \end{bmatrix}$$

$\therefore$ the eigenvector is

$$\begin{bmatrix} \frac{1}{2} - \frac{\sqrt{2}}{2}i \\ -1 \end{bmatrix}$$

$\lambda = 4 - 2\sqrt{2}\,i$

$$\begin{bmatrix} 2-(4-2\sqrt{2}\,i) & -3 \\ 4 & 6-(4-2\sqrt{2}\,i) \end{bmatrix} \rightarrow \begin{bmatrix} -2+2\sqrt{2}\,i & -3 \\ 4 & 2+2\sqrt{2}\,i \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & \frac{1}{2} + \frac{\sqrt{2}}{2}i \\ 0 & 0 \end{bmatrix}$$

$\therefore$ the eigenvector is

$$\begin{bmatrix} \frac{1}{2} + \frac{\sqrt{2}}{2}i \\ -1 \end{bmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = c_1 \begin{pmatrix} \frac{1}{2} - \frac{\sqrt{2}}{2}i \\ -1 \end{pmatrix} e^{(4+2\sqrt{2}i)t} + c_2 \begin{pmatrix} \frac{1}{2} + \frac{\sqrt{2}}{2}i \\ -1 \end{pmatrix} e^{(4-2\sqrt{2}i)t}$$