Lecture 5
Dataflow Process Models

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Philosophy of Dataflow Languages

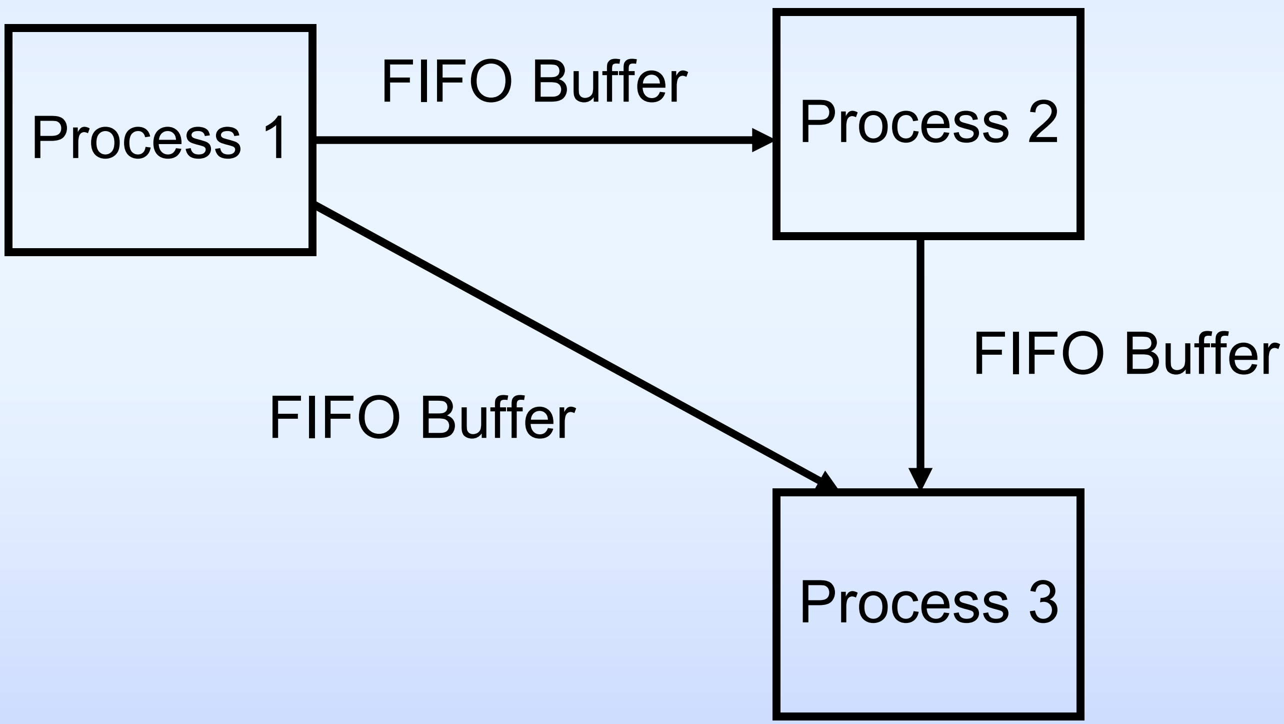
- Drastically different way of looking at computation
- Von Neumann imperative language style: program counter is king
- Dataflow language: movement of data the priority
- Scheduling responsibility of the system, not the programmer

Dataflow Languages

- Every process can run concurrently
  - Processes side-effect free resources assumed
- Processes described with imperative code
  - FSM, NDFA model of hardware or software
- Processes *only* communicate through buffers
  - Both control and data
- Parallelism is bounded by places and data-flow
  - Can describe general purpose computation this way
    - Requires alternative viewpoint and metrics
  - Fits transactional models of a system
    - Data-base (Google)
- Execution driven by demand

Dataflow Language Model

- **Processes communicating through FIFO buffers**



Dataflow Communication

- Communication *only* through buffers
  - No side effects (or shared memory)
- Buffers are unbounded for simplicity
  - Causes model complexity issues
- Token Sequence into link is sequence out of link
  - links are strictly FIFO
- Destructive read: reading a value from a buffer removes the value
  - Cannot ‘check’ to see new token without read
- Unlike shared memory, can always determine latency

Applications of Dataflow Models

- Poor fit for a word processor
  - Data-flow models are weak on control intensive behavior
- Common in signal-processing applications
  - Ordered streams of data
  - Simple map to pipelined hardware
    - Lab View, Simulink, System C Transactions
- Buffers used for signal processing applications anyway
  - FIFO buffers allow for mediation of bursty flows up to capacity of the buffer
    - Rates must strictly agree on average

Applications of Dataflow

- Good fit for block-diagram specifications
  - System Level RTL (directed links)
  - Linear/nonlinear control systems (Feedback Networks)
  - Network Computing
- Common in Electrical Engineering
- Value: reasoning about data rates, availability, latency and performance can be done abstractly
  - Used for top-level models before processes are designed
  - Allow reasoning about process requirements

Kahn Process Networks

- Proposed by Kahn in 1974 as a general-purpose scheme for parallel programming
- Laid the theoretical foundation for dataflow
- Unique attribute: deterministic

- Difficult to schedule
- Too flexible to make efficient, not flexible enough for a wide class of applications
- Never put to widespread use

Kahn Process Networks

- **Key idea:**

Reading an empty channel blocks until data is available

- **No other mechanism for sampling communication channel's contents**
  - **Can't check to see whether buffer is empty**
  - **Can't wait on multiple channels at once**

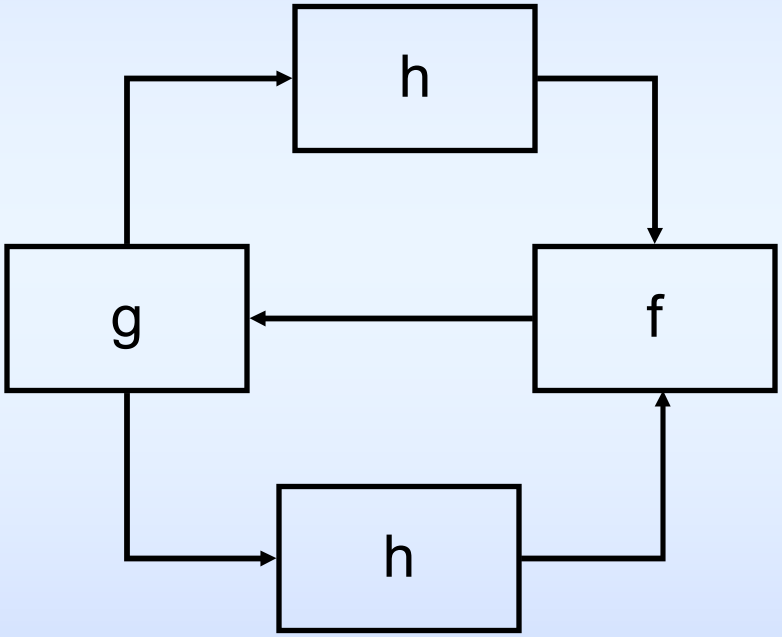
Kahn Processes

- A C-like function (Kahn used Algol)
- Arguments include FIFO channels
- Language augmented with send() and wait() operations that write and read from channels

A Kahn System

- **Prints an alternating sequence of 0's and 1's**

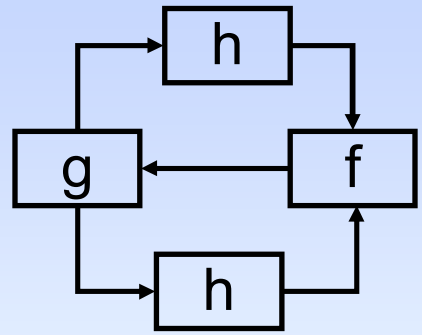
Emits a 1 then copies input to output



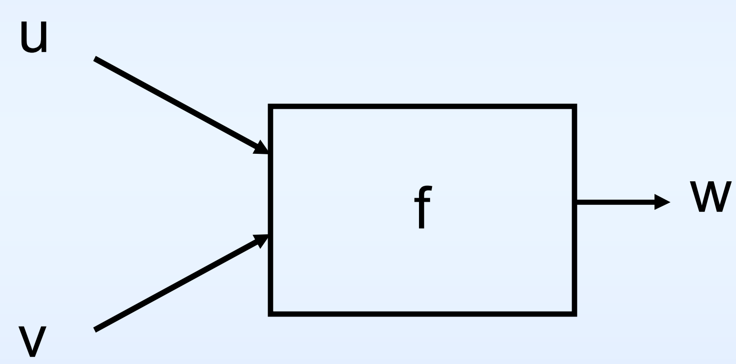
Emits a 0 then copies input to output

A Kahn Process

- **From Kahn's original 1974 paper**

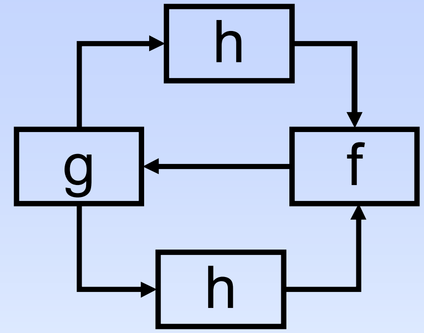



```
process f(in int u, in int v, out int w)
{
  int i; bool b = true;
  for (;;) {
   i = b ? wait(u) : wait(w);
   printf("%i\n", i);
   send(i, w);
   b = !b;
  }
}
```



Process alternately reads from u and v, prints the data value, and writes it to w

A Kahn Process



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```
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  for (;;) {
    i = b ? wait(u) : wait(w);
    printf("%i\n", i);
    send(i, w);
    b = !b;
  }
}
```
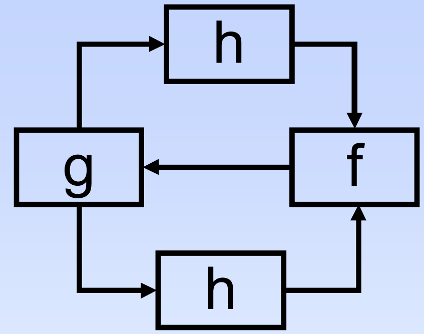

Process interface includes FIFOs

wait() returns the next token in an input FIFO, blocking if it's empty

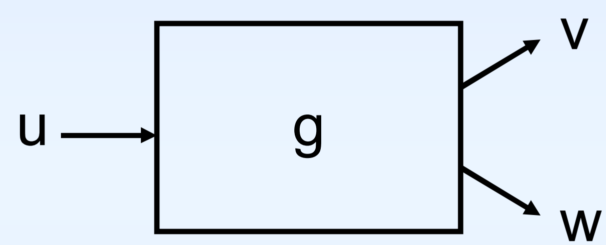
send() writes a data value on an output FIFO

A Kahn Process



- From Kahn’s original 1974 paper

```
process g(in int u, out int v, out int w)
{
  int i; bool b = true;
  for(;;) {
    i = wait(u);
    if (b) send(i, v); else send(i, w);
    b = !b;
  }
}
```

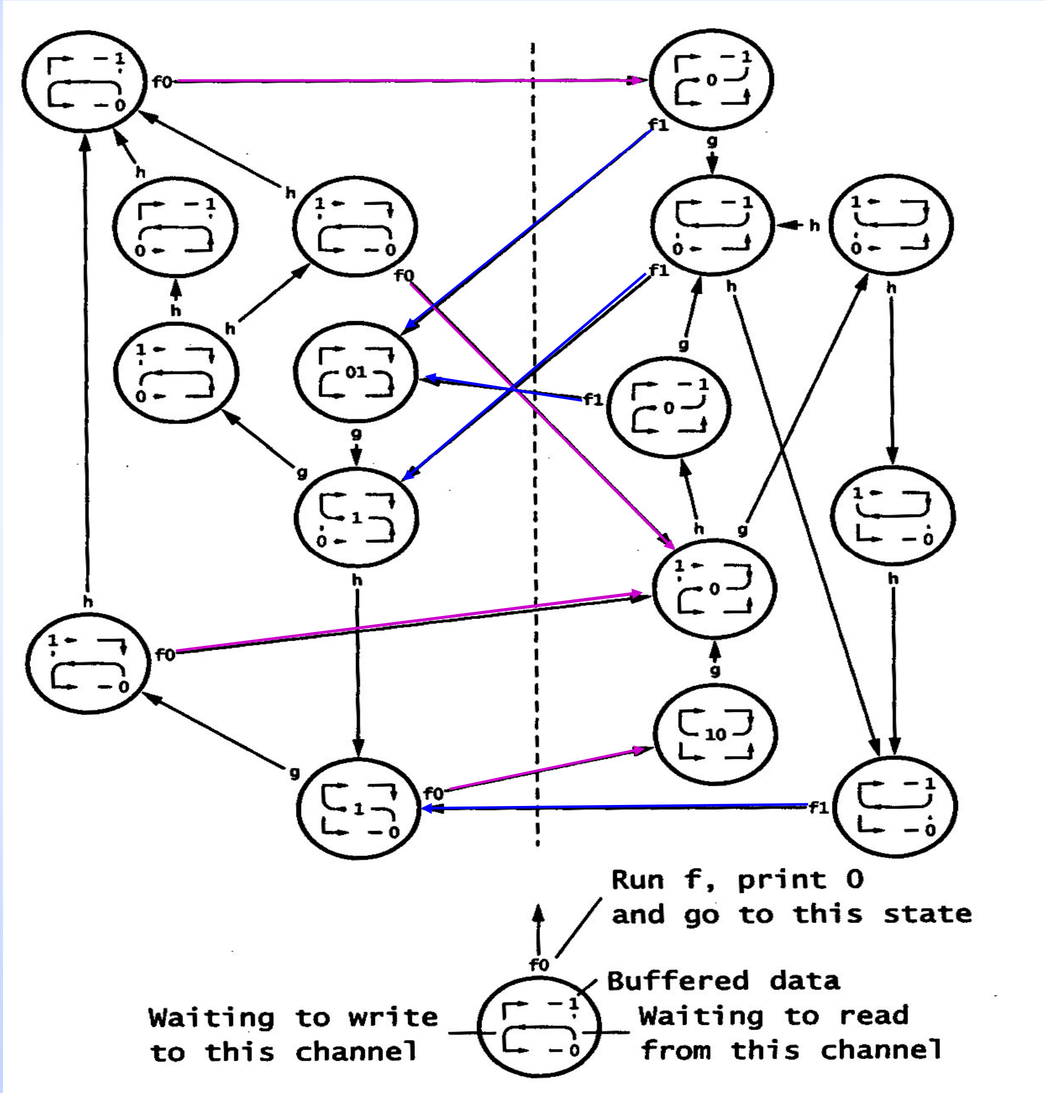


Process reads from u and alternately copies it to v and w

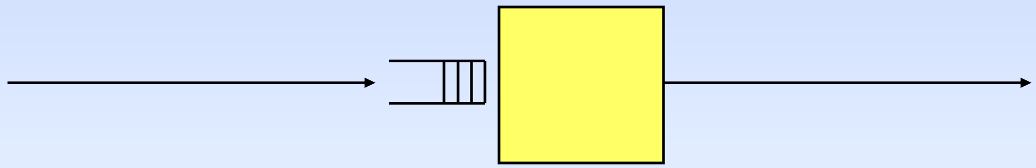
Possible Runs of Kahn System

- **Starts from upper left corner**
- **Deterministic since all output writes must cross boundary**
  - **Left going arcs '0'**
  - **Right going arcs '1'**

Thus all possible output sequences alternate 0/1/0…



Determinacy



- Process: “ordered mapping” of input sequence to output sequences
- Continuity: process uses prefix of input sequences to produce prefix of output sequences. Adding more tokens does not change the tokens already produced
- The state of each process depends on token values rather than their arrival time
- Unbounded FIFO: the speed of the two processes does not affect the sequence of data values
  - Practical networks need to mind this well

Proof of Determinism

- Because a process can’t check the contents of buffers, only read from them, each process only sees sequence of data values coming in on buffers

- Behavior of process:

Compute … read … compute … write … read … compute

- Values written only depend on program state
- Computation only depends on program state
- Reads always return sequence of data values, nothing more

Determinism

- Another way to see it:

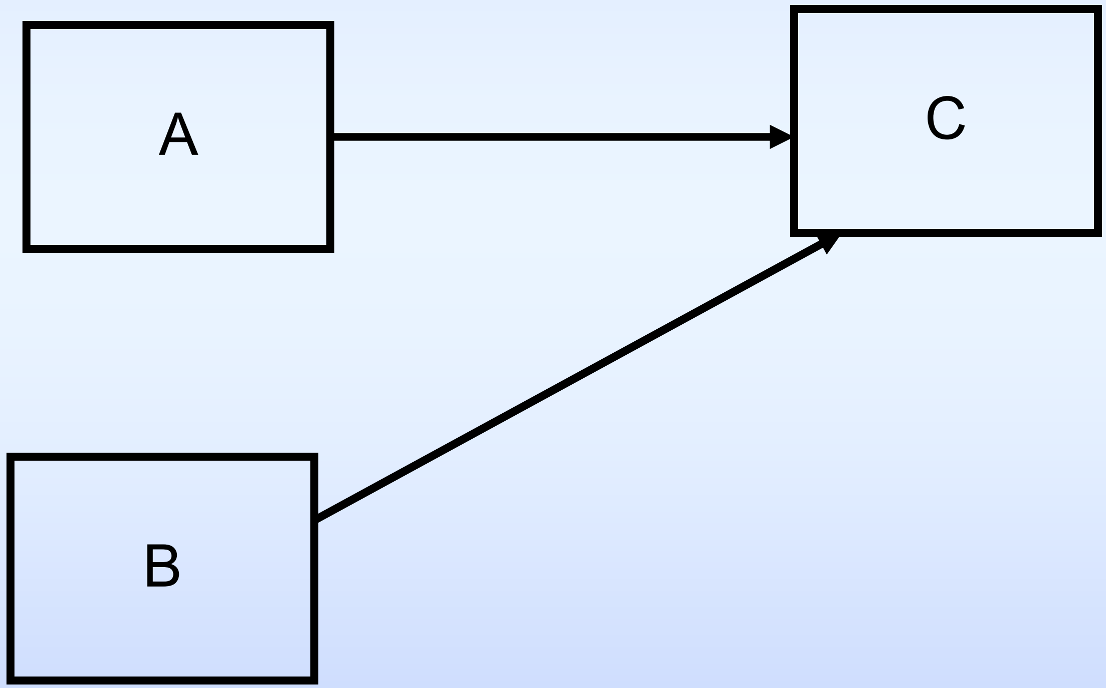
- If I'm a process, I am only affected by the sequence of tokens on my inputs
- I can't tell whether they arrive early, late, or in what order
- I will behave the same in any case
- Thus, the sequence of tokens I put on my outputs is the same regardless of the timing of the tokens on my inputs

Routes to Nondeterminism

- Allow processes to test for emptiness
  - If the token behavior changes, violates monotonic property
  - Cannot choose from possible inputs (I.e. if token on either input… is not legal)
- Allow processes themselves to be nondeterminate
- Allow more than one process to read from a channel
  - Cannot solve precedence issues in general
- Allow more than one process to write to a channel
  - Cannot fix the order of processes on channel
- Allow processes to share a variable
  - Unbounded communication bandwidth can cause several problems above…

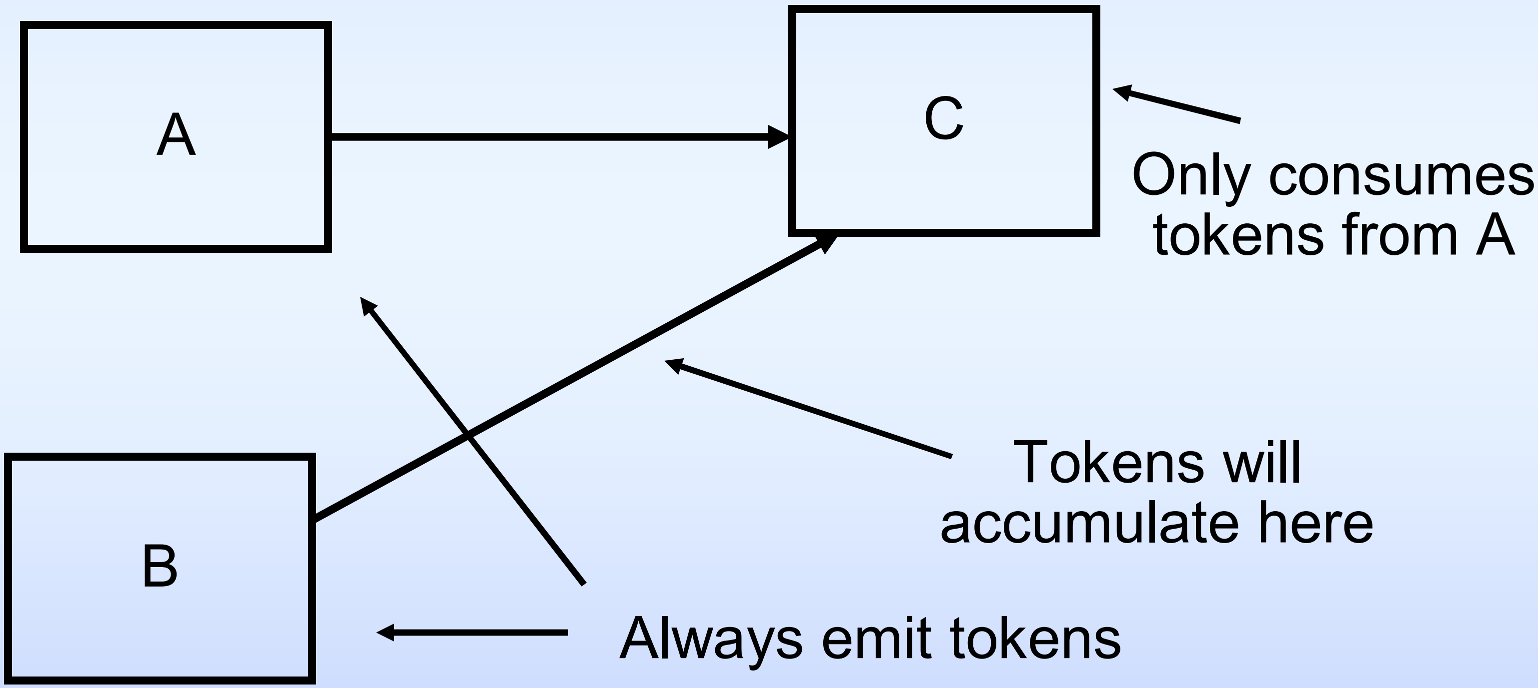
Scheduling Kahn Networks

- **Challenge is running processes without accumulating tokens**



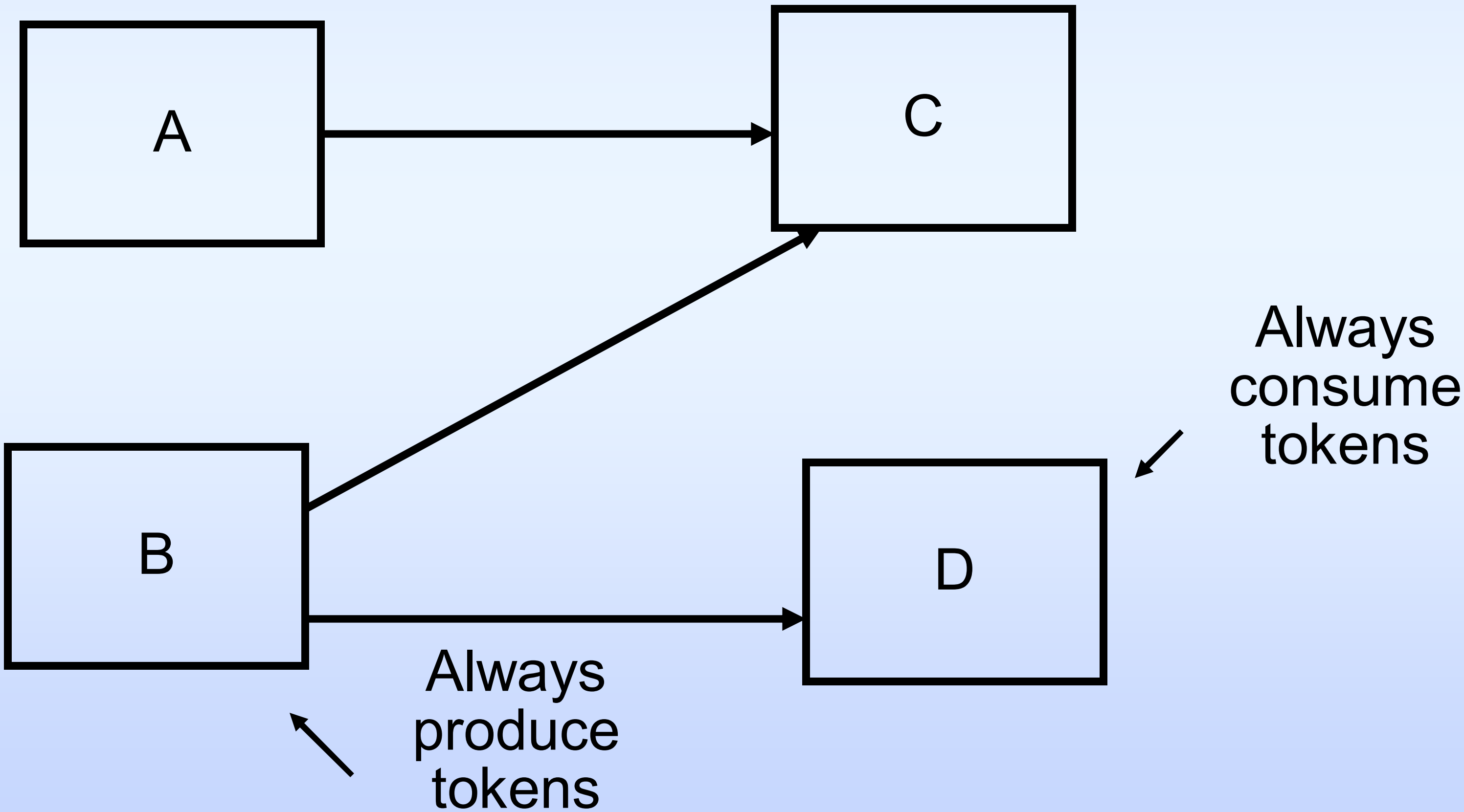
Scheduling Kahn Networks

- **Challenge is running processes without accumulating tokens**



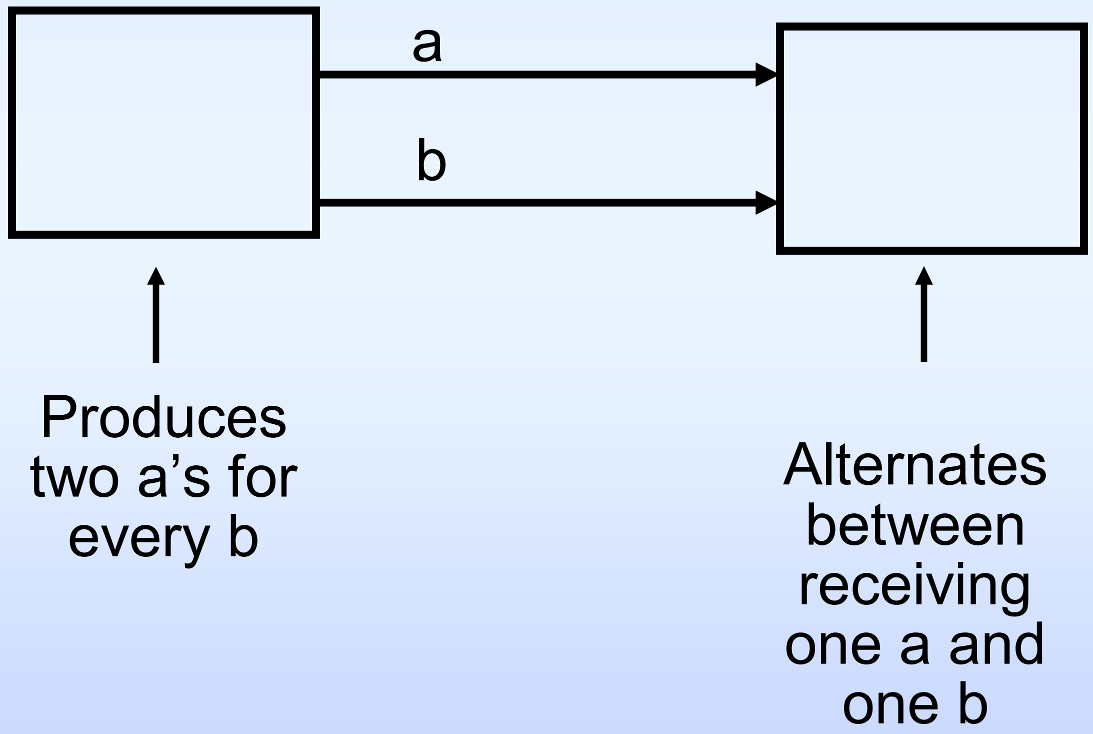
Demand-driven Scheduling?

- **Apparent solution: only run a process whose outputs are being actively solicited**
- **However...**



Other Difficult Systems

- **Not all systems can be scheduled without token accumulation**

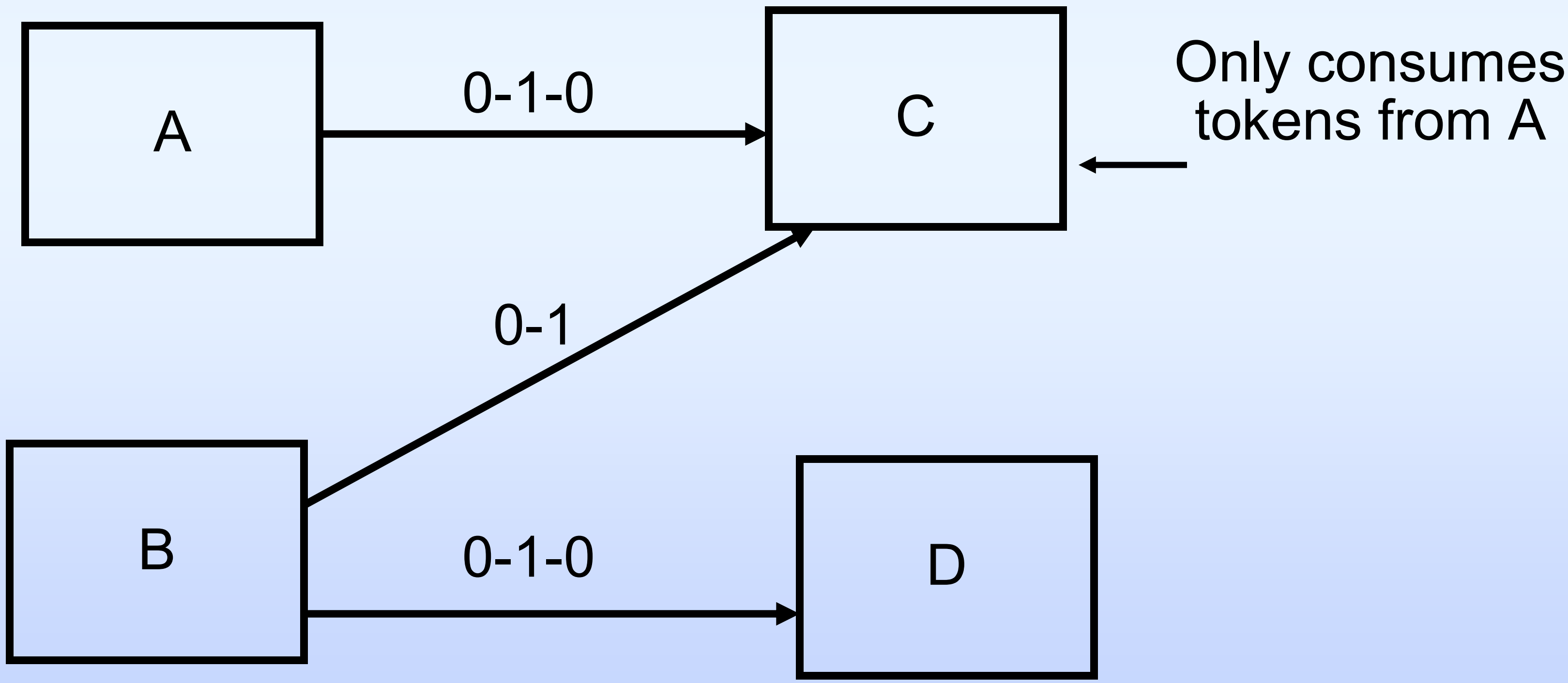


Tom Parks' Algorithm

- Schedules a Kahn Process Network in bounded memory if it is possible
- Start with bounded buffers
- Use any scheduling technique that avoids buffer overflow
- If system deadlocks because of buffer overflow, increase size of smallest buffer and continue

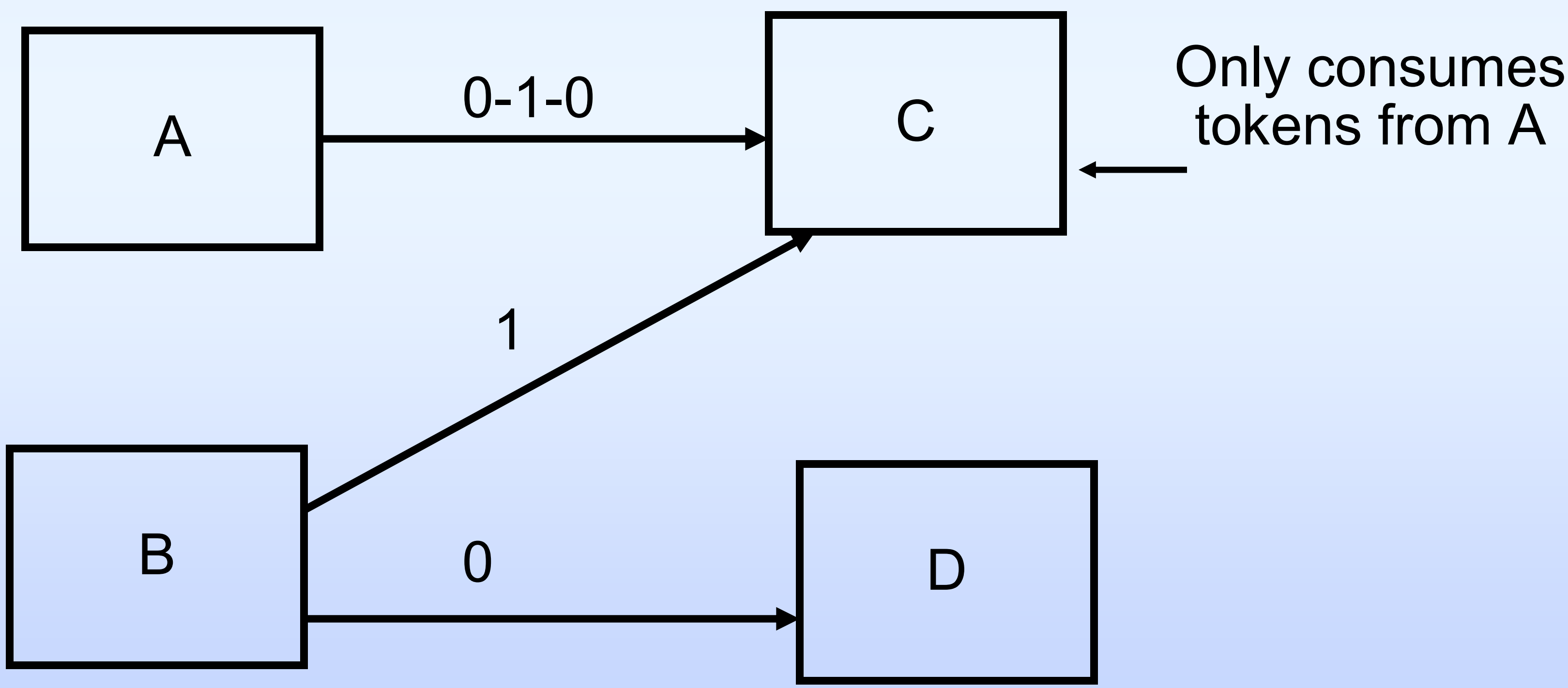
Parks' Algorithm in Action

- **Start with buffers of size 1**
- **Run A, B, C, D**



Parks' Algorithm in Action

- **B blocked waiting for space in B->C buffer**
- **Run A, then C**
- **System will run indefinitely**



Parks' Scheduling Algorithm

- Neat trick
- Whether a Kahn network can execute in bounded memory is undecidable
- Parks' algorithm does not violate this
- It will run in bounded memory if possible, and use unbounded memory if necessary

Using Parks' Scheduling Algorithm

- It works, but…

- Requires dynamic memory allocation
- Does not guarantee minimum memory usage
- Scheduling choices may affect memory usage
- Data-dependent decisions may affect memory usage
- Relatively costly scheduling technique
- Detecting deadlock may be difficult

Kahn Process Networks

- Their beauty is that the scheduling algorithm does not affect their functional behavior
- Difficult to schedule because of need to balance relative process rates
- System inherently gives the scheduler few hints about appropriate rates
- Parks' algorithm expensive and fussy to implement
- Might be appropriate for coarse-grain systems
  - Scheduling overhead dwarfed by process behavior

Synchronous Dataflow (SDF)

- **Edward Lee and David Messerchmitt, Berkeley, 1987**

- **Restriction of Kahn Networks to allow compile-time scheduling**
- **Basic idea: each process reads and writes a fixed number of tokens each time it fires:**

loop

read 3 A, 5 B, 1 C … compute … write 2 D, 1 E, 7 F

end loop

Operational Semantics
Firing Rule

- **Tokens → Data**
- **Assignment → Placing a token in the output arc**
- **Snapshot / configuration: state**
- **Computation**
  - **The intermediate step between snapshots / configurations**
- **An actor of a dataflow graph is enabled if there is a token on each of its input arcs**

Synchronous Dataflow (SDF)

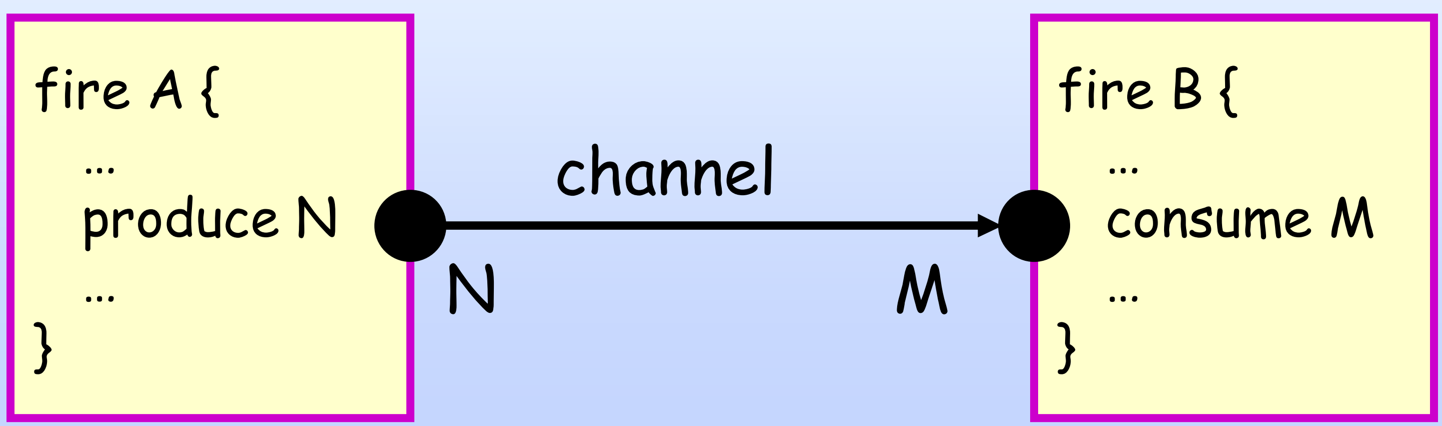
Fixed Production/Consumption Rates

- **Balance equations (one for each channel):**

$$f_A N = f_B M$$

- M: number of tokens consumed
- f_A, f_B: number of firings per "iteration"
- N: number of tokens produced

- **Schedulable statically**
- **Get a well-defined "iteration"**
- **Decidable:**
  - **buffer memory requirements**
  - **deadlock**

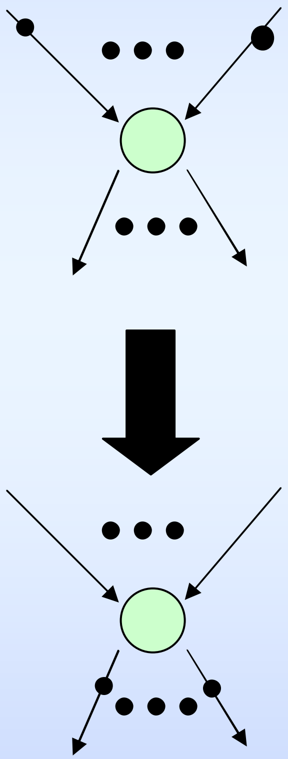


SDF and Signal Processing

- Restriction natural for multirate signal processing
- Typical signal-processing processes:
- Unit-rate
  - Adders, multipliers
- Upsamplers (1 in, n out)
- Downsamplers (n in, 1 out)

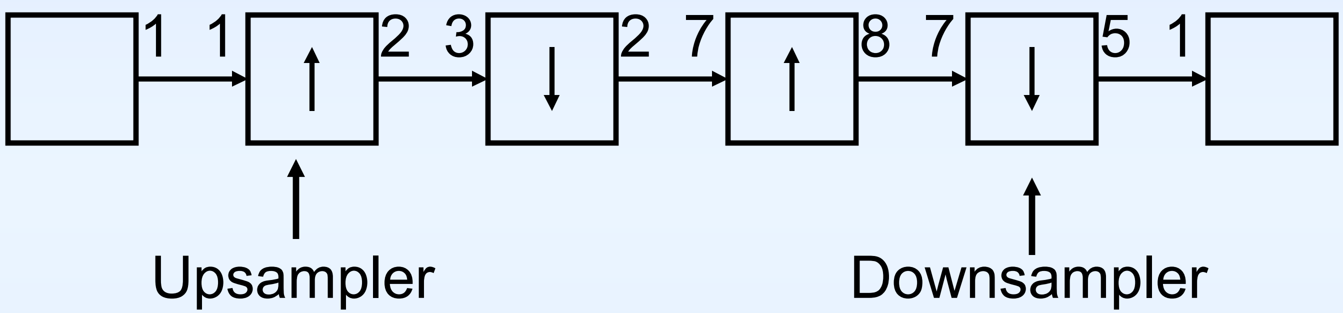
Operational Semantics
Firing Rule

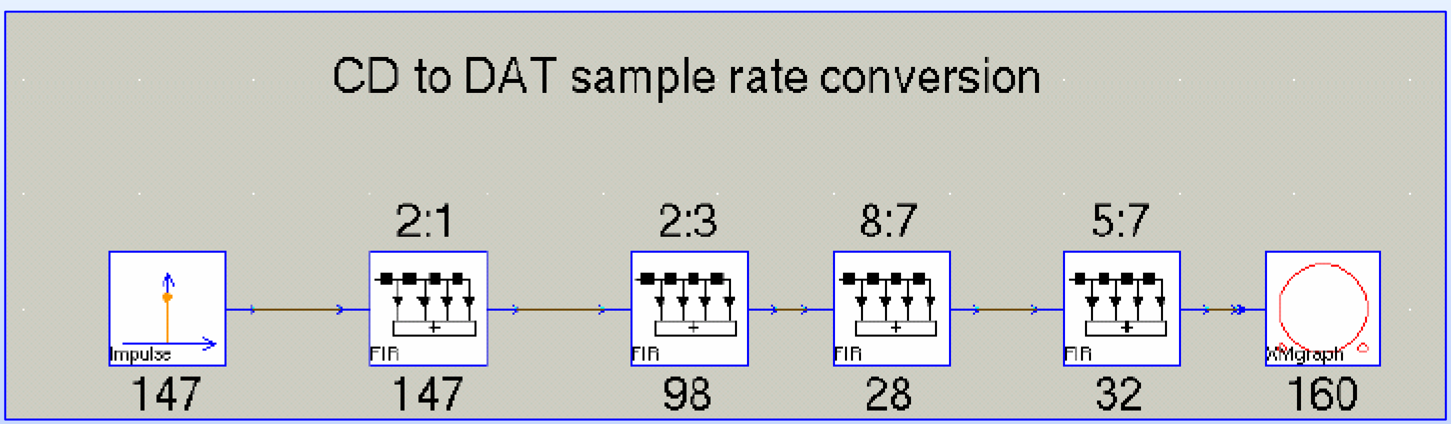
- **Any enabled actor may be fired to define the "next state" of the computation**
- **An actor is fired by removing a token from each of its input arcs and placing tokens on each of its output arcs.**
- **Computation ➔ A Sequence of Snapshots**
  - **Many possible sequences as long as firing rules are obeyed**
  - **Determinacy**
  - **"Locality of effect"**



Multi-rate SDF System

- **DAT-to-CD rate converter**
- **Converts a 44.1 kHz sampling rate to 48 kHz**



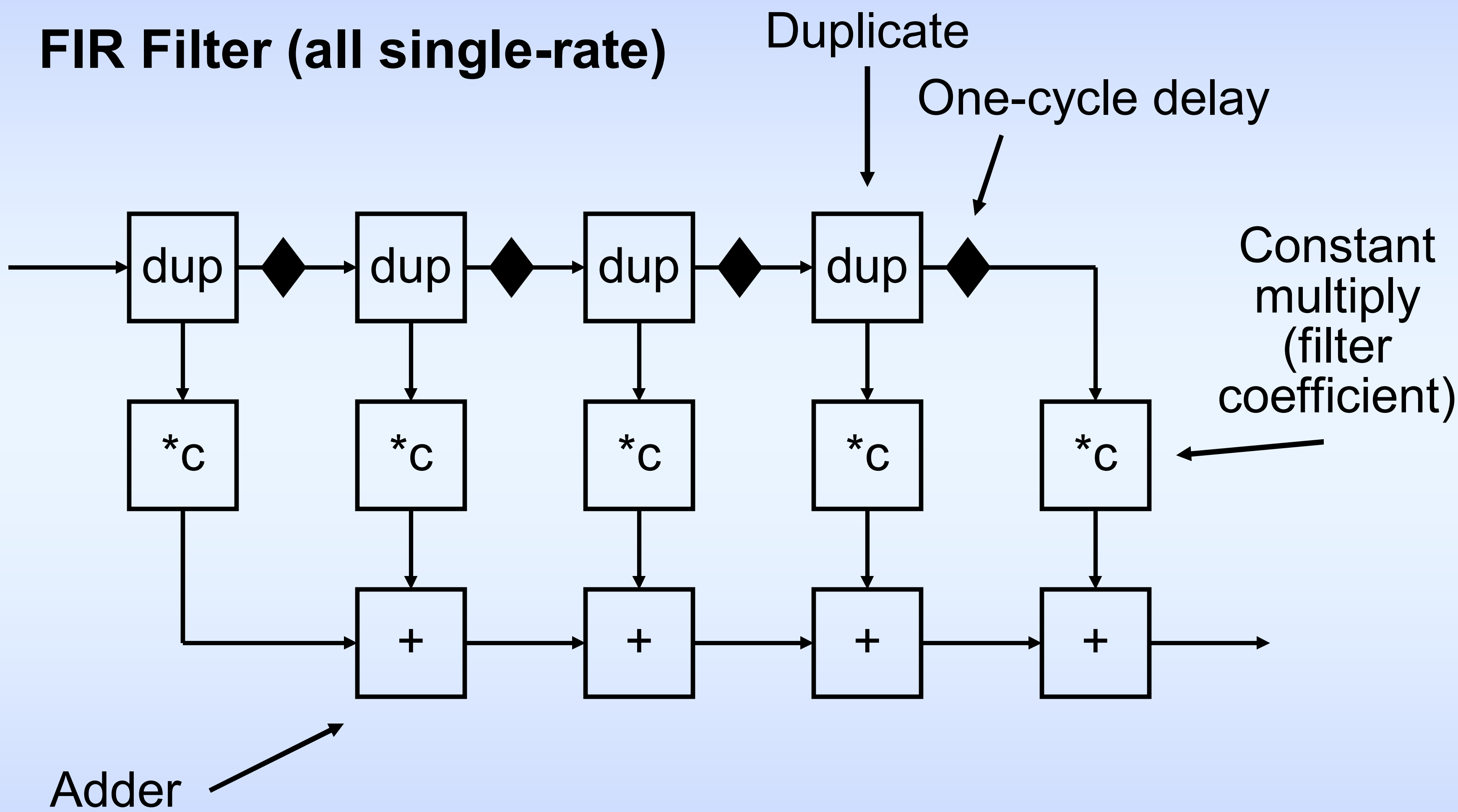


Delays

- Kahn processes often have an initialization phase
- SDF doesn’t allow this because rates are not always constant
- Alternative: an SDF system may start with tokens in its buffers
- These behave like delays (signal-processing)
- Delays are sometimes necessary to avoid deadlock

Example SDF System

- **FIR Filter (all single-rate)**



SDF Scheduling

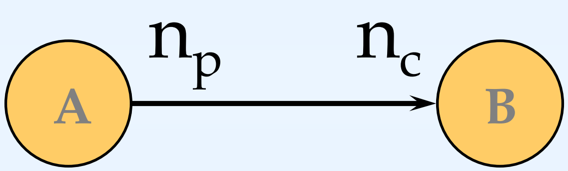
- Schedule can be determined completely before the system runs

- Two steps:

1. Establish relative execution rates by solving a system of linear equations
2. Determine periodic schedule by simulating system for a single round

Balance equations

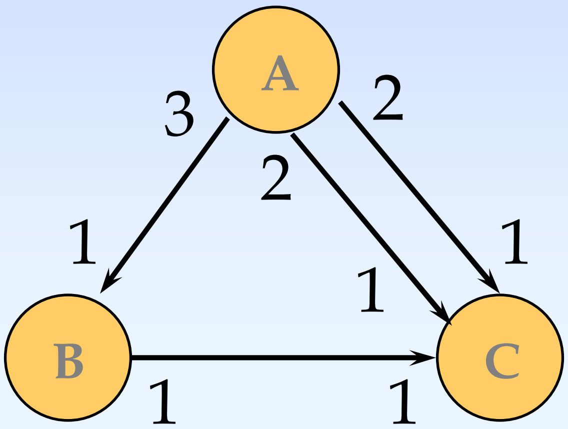
- Number of produced tokens must equal number of consumed tokens on every edge



- Repetitions (or firing) vector v_S of schedule S: number of firings of each actor in S
- $v_S(A)\ n_p = v_S(B)\ n_c$

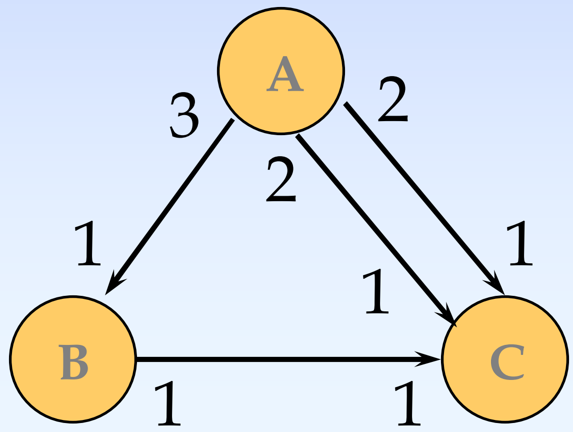
  must be satisfied for each edge

Balance equations



- Balance for each edge:
  - $3\ v_S(A) - v_S(B) = 0$
  - $v_S(B) - v_S(C) = 0$
  - $2\ v_S(A) - v_S(C) = 0$
  - $2\ v_S(A) - v_S(C) = 0$

Balance equations



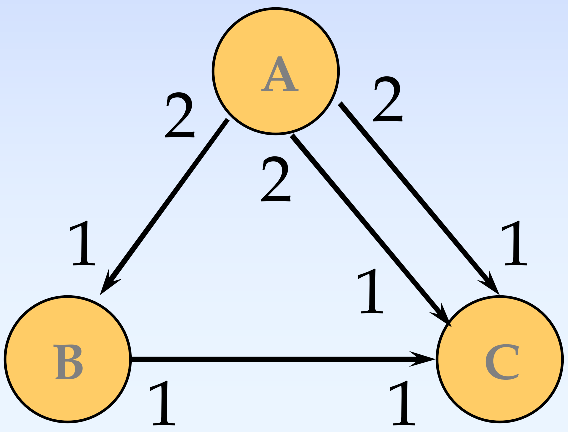
$$M = \begin{vmatrix} 3 & -1 & 0 \\ 0 & 1 & -1 \\ 2 & 0 & -1 \\ 2 & 0 & -1 \end{vmatrix}$$

- $M\ v_S = 0$

  iff S is periodic
- Full rank (as in this case)
  - no non-zero solution
  - no periodic schedule

(too many tokens accumulate on A->B or B->C)

Balance equations



$$M = \begin{vmatrix} 2 & -1 & 0 \\ 0 & 1 & -1 \\ 2 & 0 & -1 \\ 2 & 0 & -1 \end{vmatrix}$$

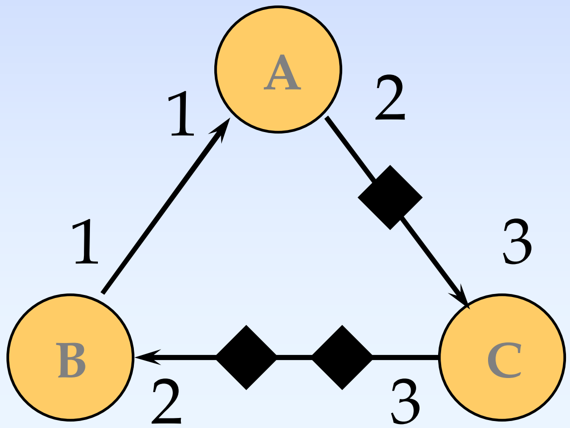
- Non-full rank
  - infinite solutions exist (linear space of dimension 1)
- Any multiple of $q = |1 \quad 2 \quad 2|^T$ satisfies the balance equations
- ABCBC and ABBCC are minimal valid schedules
- ABABBCBCCC is non-minimal valid schedule

Static SDF scheduling

- Main SDF scheduling theorem (Lee '86):
  - A connected SDF graph with *n* actors has a periodic schedule iff its topology matrix M has rank *n-1*
  - If M has rank *n-1* then there exists a unique smallest integer solution q to

    M q = 0
- Rank must be at least *n-1* because we need at least *n-1* edges (connected-ness), providing each a linearly independent row
- Admissibility is not guaranteed, and depends on initial tokens on *cycles*

Admissibility of schedules



- No admissible schedule:

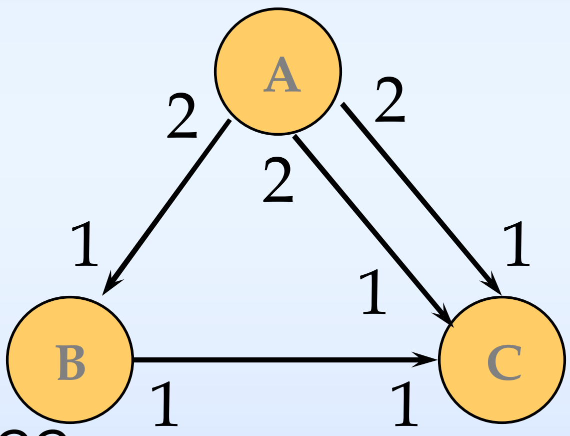
  BACBA, then deadlock…
- Adding one token on A->C makes

  BACBACBA valid
- Making a periodic schedule admissible is always possible, but changes specification...

From repetition vector to schedule

- Repeatedly schedule fireable actors up to number of times in repetition vector

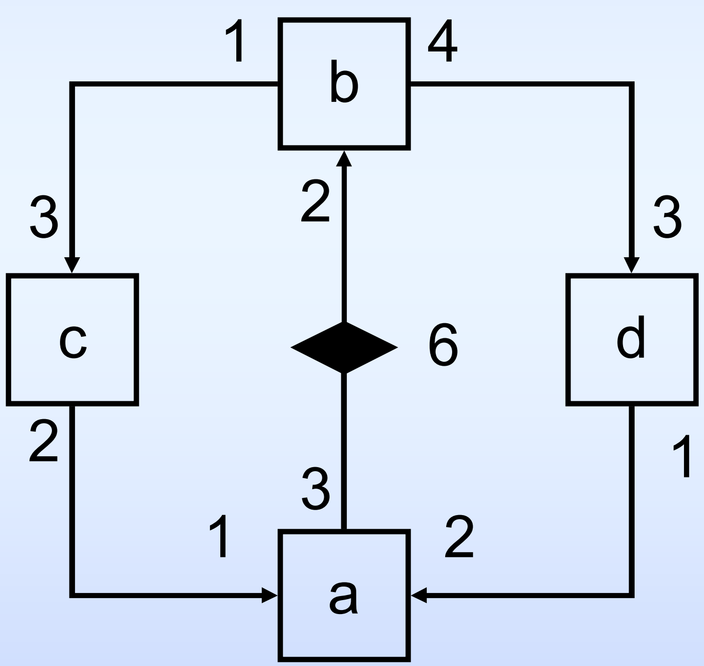
  $q = |1 \quad 2 \quad 2|^T$



- Can find either ABCBC or ABBCC
- If deadlock before original state, no valid schedule exists (Lee ‘86)

Calculating Rates

- **Each arc imposes a constraint**



$3a - 2b = 0$

$4b - 3d = 0$

$b - 3c = 0$

$2c - a = 0$

$d - 2a = 0$

Solution?

$a = 2c$

$b = 3c$

$d = 4c$

Calculating Rates

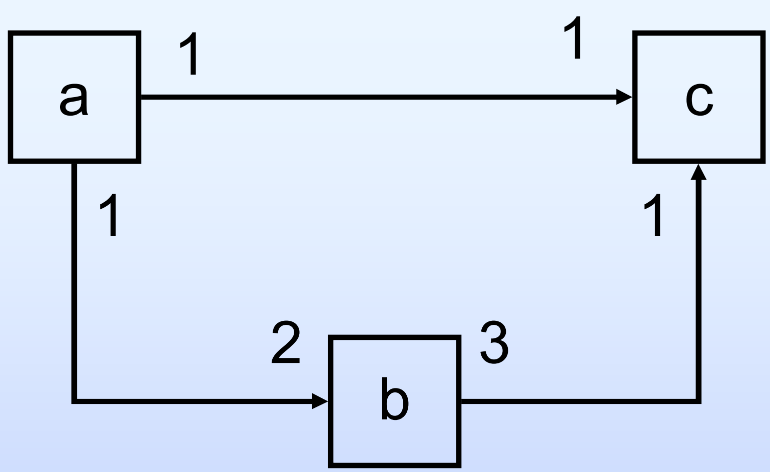
- Consistent systems have a one-dimensional solution
  - Usually want the smallest integer solution

- Inconsistent systems only have the all-zeros solution

- Disconnected systems have two- or higher-dimensional solutions

An Inconsistent System

- **No way to execute it without an unbounded accumulation of tokens**
- **Only consistent solution is "do nothing"**



$$a - c = 0$$

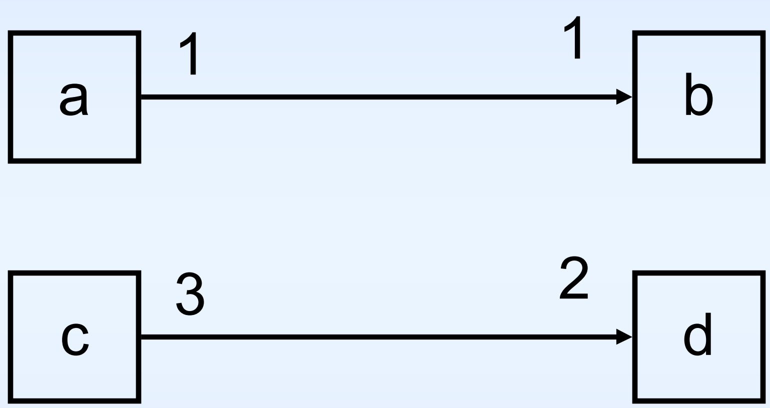
$$a - 2b = 0$$

$$3b - c = 0$$

$$3a - 2c = 0$$

An Underconstrained System

- **Two or more unconnected pieces**
- **Relative rates between pieces undefined**

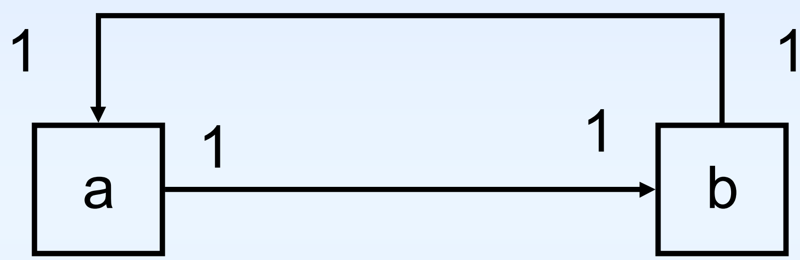


$$a - b = 0$$

$$3c - 2d = 0$$

Consistent Rates Not Enough

- **A consistent system with no schedule**
- **Rates do not avoid deadlock**



- **Solution here: add a delay on one of the arcs**

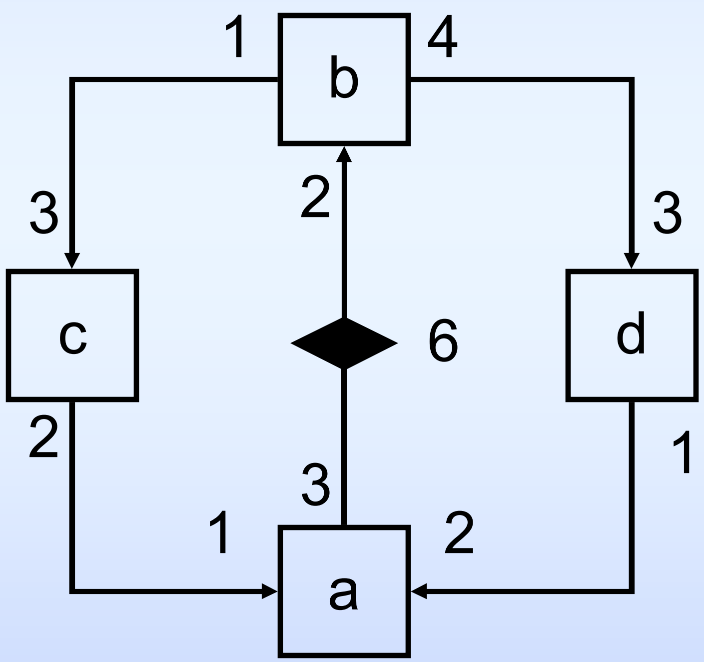
SDF Scheduling

- **Fundamental SDF Scheduling Theorem:**

  If rates can be established, any scheduling algorithm that avoids buffer underflow will produce a correct schedule if it exists

Scheduling Example

- **Theorem guarantees any valid simulation will produce a schedule**



a=2 b=3 c=1 d=4

Possible schedules:

BBBCDDDDAA

BDBDBCADDA

BBDDBDDCAA

… many more

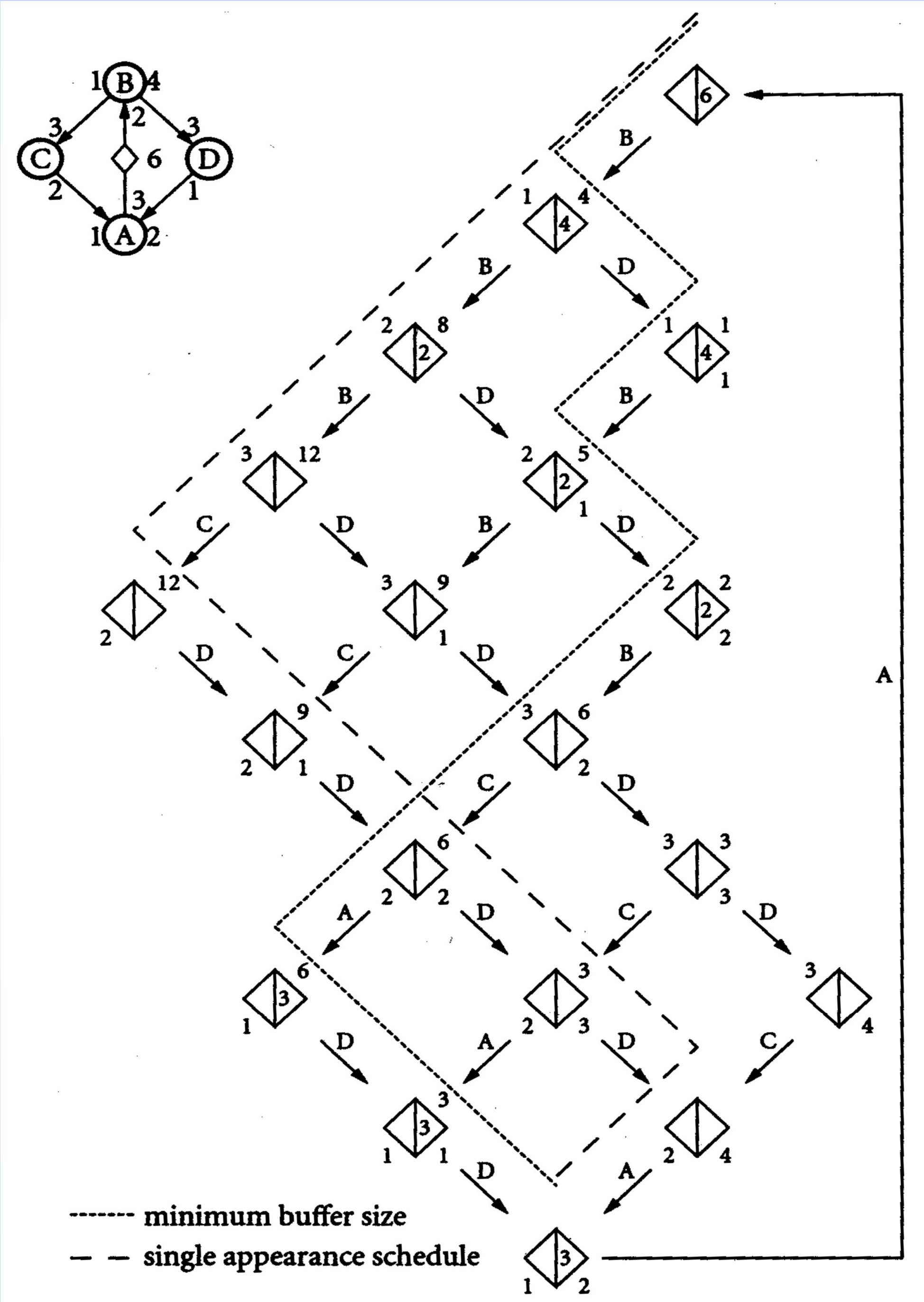
BC … is not valid

SDF Scheduling

- Goal: a sequence of process firings that
- Runs each process at least once in proportion to its rate
- Avoids underflow
  - no process fired unless all tokens it consumes are available
- Returns the number of tokens in each buffer to their initial state
- Result: the schedule can be executed repeatedly without accumulating tokens in buffers

Schedules

- Dash is single appearance schedule
- Short Dash is minimum buffer schedule
- Note: SDF schedules form a lattice



Scheduling Choices

- SDF Scheduling Theorem guarantees a schedule will be found if it exists
- Systems often have many possible schedules
- How can we use this flexibility?
  - Reduced code size
  - Reduced buffer sizes

SDF Code Generation

- Often done with prewritten blocks
- For traditional DSP, handwritten implementation of large functions (e.g., FFT)
- One copy of each block’s code made for each appearance in the schedule
  - I.e., no function calls

Code Generation

- In this simple-minded approach, the schedule

BBBCDDDDAA

would produce code like

```
B;
B;
C;
D;
D;
D;
D;
A;
A;
```

Looped Code Generation

- Obvious improvement: use loops

- Rewrite the schedule in "looped" form:

  (3 B) C (4 D) (2 A)

- Generated code becomes

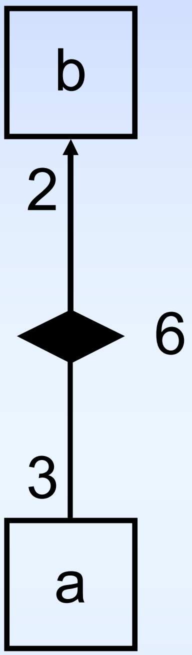
```
for ( i = 0 ; i < 3; i++) B;
C;
for ( i = 0 ; i < 4 ; i++) D;
for ( i = 0 ; i < 2 ; i++) A;
```

Single-Appearance Schedules

- Often possible to choose a looped schedule in which each block appears exactly once
- Leads to efficient block-structured code
  - Only requires one copy of each block's code
- Does not always exist
- Often requires more buffer space than other schedules

Finding Single-Appearance Schedules

- **Always exist for acyclic graphs**
  - **Blocks appear in topological order**
- **For SCCs, look at number of tokens that pass through arc in each period (follows from balance equations)**
- **If there is at least that much delay, the arc does not impose ordering constraints**
- **Idea: no possibility of underflow**



a=2 b=3

6 tokens cross the arc

delay of 6 is enough

Finding Single-Appearance Schedules

- **Recursive strongly-connected component decomposition**

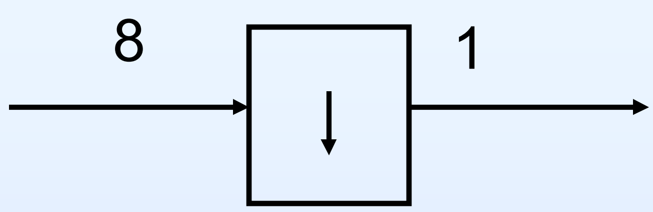
- **Decompose into SCCs**
- **Remove non-constraining arcs**
- **Recurse if possible**
  - **Removing arcs may break the SCC into two or more**

Minimum-Memory Schedules

- Another possible objective
- Often increases code size (block-generated code)
- Static scheduling makes it possible to exactly predict memory requirements
- Simultaneously improving code size, memory requirements, sharing buffers, etc. remain open research problems

Cyclo-static Dataflow

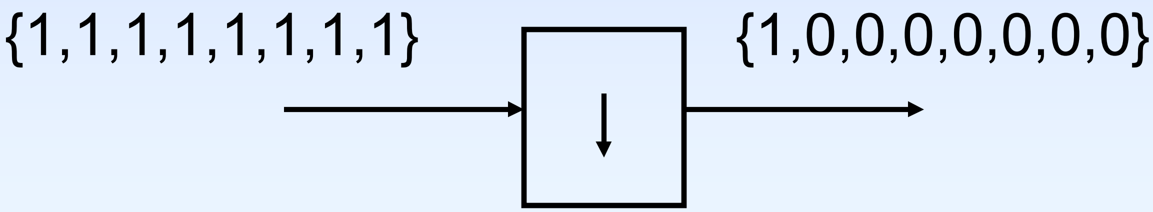
- **SDF suffers from requiring each process to produce and consume all tokens in a single firing**
- **Tends to lead to larger buffer requirements**
- **Example: downsampler**



- **Don't really need to store 8 tokens in the buffer**
- **This process simply discards 7 of them, anyway**

Cyclo-static Dataflow

- **Alternative: have periodic, binary firings**



- **Semantics: first firing: consume 1, produce 1**
- **Second through eighth firing: consume 1, produce 0**

Cyclo-Static Dataflow

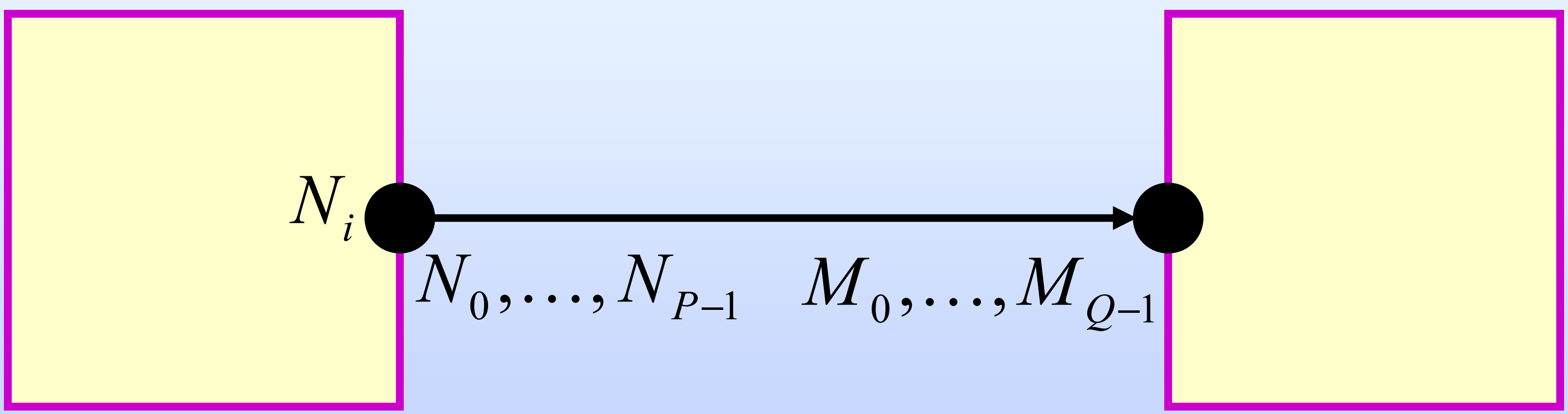
- Scheduling is much like SDF
- Balance equations establish relative rates as before
- Any scheduler that avoids underflow will produce a schedule if one exists
- Advantage: even more schedule flexibility
- Makes it easier to avoid large buffers
- Especially good for hardware implementation:
  - Hardware likes moving single values at a time

Cyclostatic Dataflow (CSDF)

(Lauwereins et al., TU Leuven, 1994)

- **Actors cycle through a regular production/consumption pattern.**
- **Balance equations become:**

$$f_A \sum_{i=0}^{R-1} N_{i \bmod P} = f_B \sum_{i=0}^{R-1} M_{i \bmod Q}; \; R = lcm(P,Q)$$

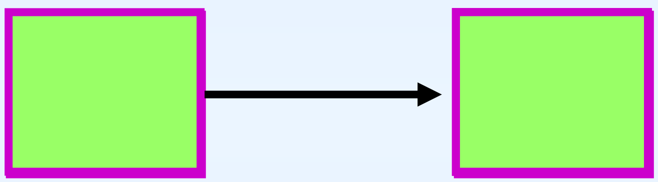


Cyclo-Static Dataflow

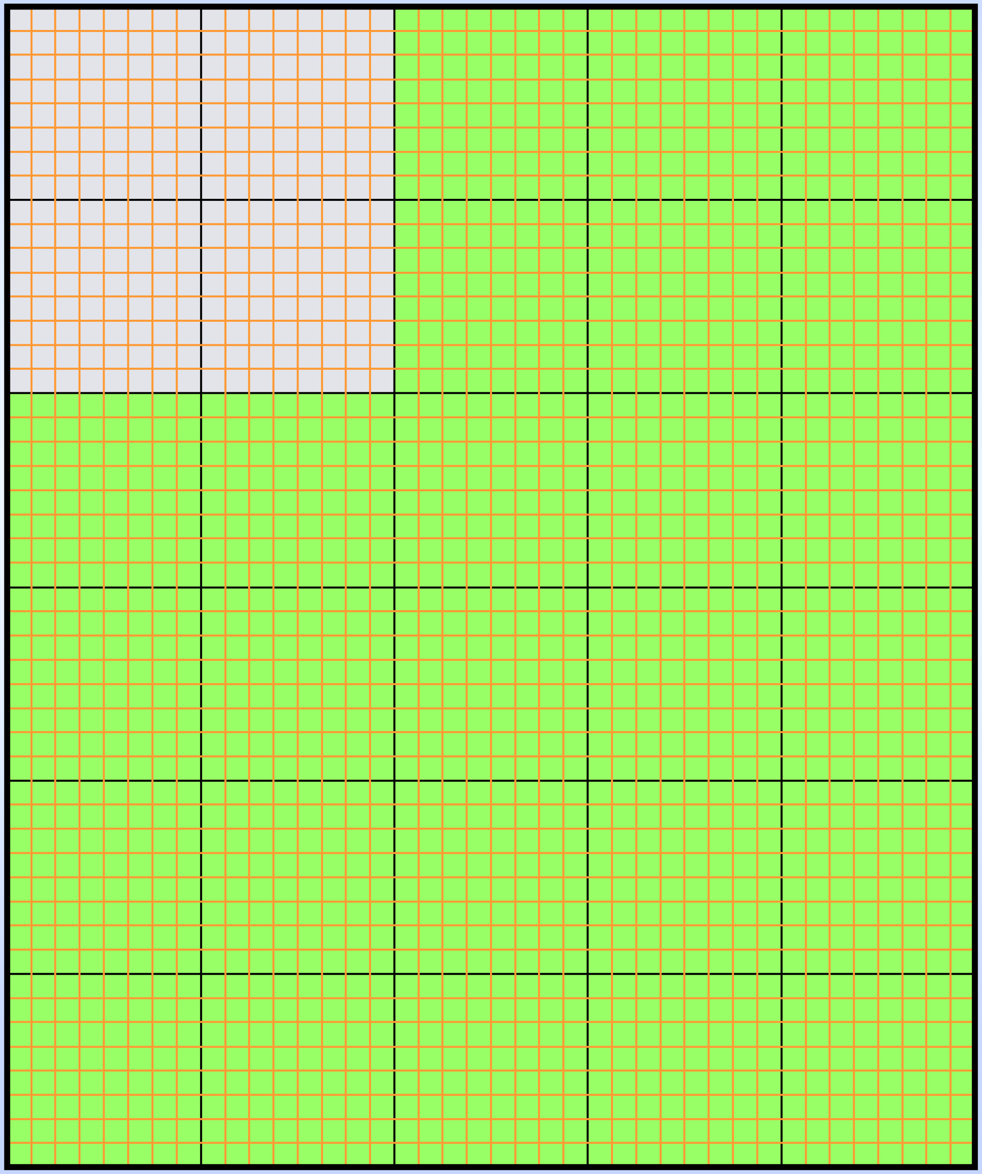
- Scheduling similar to SDF
- Balance equations establish relative rates
- Key: avoid underflow of channel
- Advantages
  - Increased schedule flexibility
    - Easier to avoid large buffers
  - Closer to parallel hardware model
    - Links move single values at a time

Multidimensional SDF
(Lee, 1993)

- **Production and consumption of *N*-dimensional arrays of data:**



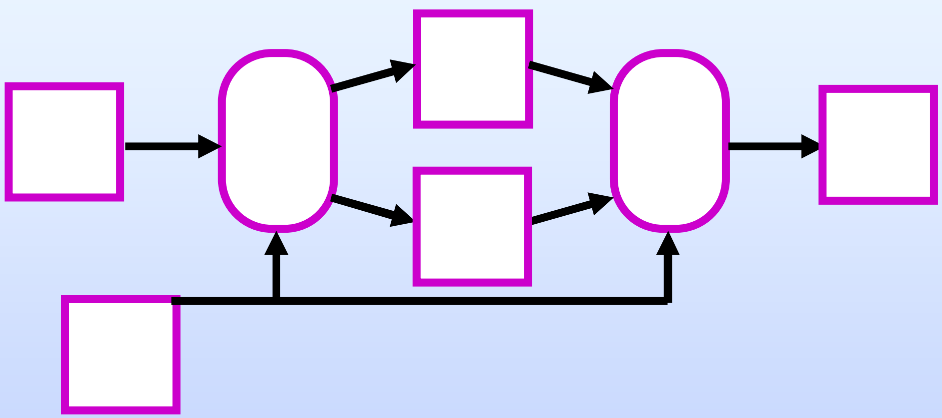
- **Balance equations and scheduling policies generalize.**
- **Much more data parallelism is exposed.**



Boolean and Integer Dataflow (BDF, IDF)

(Lee and Buck, 1993)

- **Balance equations are solved symbolically in terms of unknowns that become known at run time.**
- **An *annotated schedule* is constructed with predicates guarding each action.**
- **Existence of such an annotated schedule is undecidable (as is deadlock & bounded memory)**
  - **However often can check efficiently**



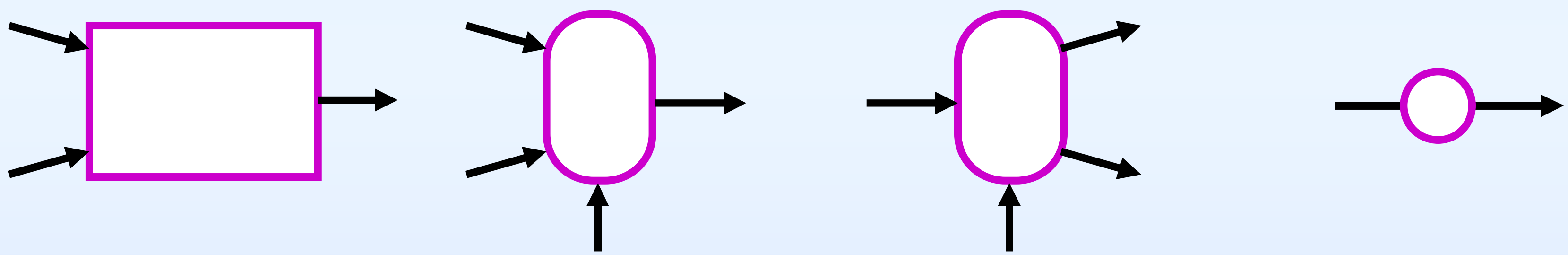
$$f_{switch} b = f_B$$

$$f_{switch}(1-b) = f_C$$

...

Undecidability
(Buck '93)

- **Sufficient set of actors for undecidability:**
  - **boolean functions on boolean tokens**
  - **switch and select**
  - **initial tokens on arcs**



- **Undecidable:**
  - **deadlock**
  - **bounded buffer memory**
  - **existence of an annotated schedule**

Dynamic Dataflow (DDF)

- Actors have *firing rules*
  - Data consumed/produced may vary depending on the values
  - Set of finite prefixes on input sequences
  - Firing function applied to finite prefixes yield finite outputs
- Scheduling objectives:
  - Do not stop if there are executable actors
  - Execute in bounded memory if this is possible
  - Maintain determinacy if possible
- Policies that fail:
  - Data-driven execution
  - Demand-driven execution
  - Fair execution
  - Many balanced data/demand-driven strategies
- Policy that succeeds (Parks 1995):
  - Execute with bounded buffers
  - Increase bounds only when deadlock occurs

Summary of Dataflow

- Processes communicating exclusively through FIFOs

- Kahn process networks
  - Blocking read, nonblocking write
  - Deterministic
  - Hard to schedule
  - Parks' algorithm requires deadlock detection, dynamic buffer-size adjustment

Summary of Dataflow

- Synchronous Dataflow (SDF)
- Firing rules:
  - Fixed token consumption/production
- Can be scheduled statically
  - Solve balance equations to establish rates
  - Any correct simulation will produce a schedule if one exists
- Looped schedules
  - For code generation: implies loops in generated code
  - Recursive SCC Decomposition
- CSDF: breaks firing rules into smaller pieces
  - Scheduling problem largely the same