HW #4 Solutions

1) NMOS:

	0.45 μm	0.90 μm (2X)	2.25 μm (5X)	4.5 μ (10X)
$C_{GDO} = 0.8\,fF/\mu m \cdot W$	0.36 fF	0.720 fF	1.8 fF	3.6 fF
$C_{db} = K_{eqn} A_{dn} C_j = (K_{eqn}\, 0.5\mu m \cdot W \cdot 1 fF)$	K_{eqn} 0.225 fF	K_{eqn} 0.45 fF	K_{eqn} 1.125 fF	K_{eqn} 2.25
$C_g = C_{ox} W_n L_n = \frac{\epsilon_{ox}}{t_{ox}} (W_n)(0.18\mu m)$	0.704 fF	1.418 fF	3.544 fF	7.09 fF
$C_{dsw} = K_{eqnsw} P_{sw} C_{jsw}$	K_{sw} 0.348 fF	K_{eqsw} 0.456 fF	K_{eqsw} 0.78 fF	K_{eqsw} 1.32 fF
$R_s = R_s = \frac{L}{W} \cdot r_{sh}$	2.8 Ω	1.4 Ω	0.56 Ω	0.28 Ω

PMOS:

C_{GDO}	0.292 fF	0.585 fF	1.463 fF	2.925 fF
$C_{db} = K_{eqp}(0.5\mu m \cdot W \cdot 1.2 fF)$	K_p 0.270 fF	K_p 0.540 fF	K_p 1.35 fF	K_p 2.7 fF
$C_g = C_{ox} W_p L_p$	0.708 fF	1.418 fF	3.544 fF	7.09 fF
$C_{dsw} = K_{eqpsw} P_{sw} C_{jsw}$	K_{psw} 0.275 fF	K_{psw} 0.361 fF	K_{psw} 0.617 fF	K_{psw} 1.05 fF
$R_s = R_D = \frac{L}{W} \cdot r_{sh}$	2.8 Ω	1.4 Ω	0.56 Ω	0.28 Ω

2) $K_{eq} = \frac{-\phi_0^m}{(V_H - V_L)(1-m)} \left[(\phi_0 - V_H)^{1-m} - (\phi_0 - V_{low})^{1-m} \right]$

‾|_ nmos: $V_H = -1.8$ $V_L = -0.9$ — $K_{eq} = 0.696$, $K_{eqsw} = 0.863$

pmos: — $K_{eq} = 0.849$, $K_{eqsw} = 0.869$

_|‾ pmos: $V_H = -1.8$ $V_L = -0.9$ — $K_{eq} = 0.678$, $K_{eqsw} = 0.717$

nmos: $V_H = -0.9$ $V_L = 0$ — $K_{eq} = 0.847$, $K_{eqsw} = 0.934$

PMOS w/ $W = 1.0\mu m$:
$C_{GDO} = 0.649\,fF$
$C_{db} = K_{eqp}\, 0.6\,fF$
$C_L = 1.5733\,fF$
$C_{dsw} = K_{eqsw}\, 0.38\,fF$

_|‾ $\Rightarrow C = C_n + C_p = 1.944 + 3.546 = 5.49\,fF$

‾|_ $\Rightarrow C = C_n + C_p = 1.885 + 3.71 = 5.596\,fF$

$\Delta t \approx C \frac{\Delta V}{I_{avg}}$

⇒ Find average currents

_|‾ ⇒ PMOS Saturated ⇒ then linear

@ out = 0V: $I = K_p' \left(V_{GT} V_{min} - \frac{V_{min}^2}{2} \right) \frac{W}{L}$

$I = 36\mu A ((1.8-0.5)^2 \tfrac{1}{2}) = 30.42\,\mu A \left(\frac{1}{0.18}\right)$

out = 1.25V: $I = 36\mu A ((1.8-0.5)1.25 - \frac{1.25^2}{2}) = 30.37\,\mu A \left(\frac{1}{0.18}\right)$

} $I_{avg} = 168.83\,\mu A$

‾|_ ⇒ NMOS saturated ⇒ then linear

$I = 180\mu A/V^2 \frac{W}{L} ((1.8-0.5)0.4 - \frac{0.4^2}{2}) \Rightarrow 79.2\,\mu A \frac{0.45}{0.18}$ } $I_{avg} = 192\,\mu A$

out = 1.25V ⇒ same current

$t_{plh} \simeq 5.49\,fF\left(\frac{0.9V}{168.83\mu A}\right)$

$= 29.27$ ps

$t_{phl} \simeq 5.596\,fF\left(\frac{0.9V}{192\mu A}\right)$

$\simeq 26.23$ ps

V_M: Assum both transistors are in normal saturation:

$$\frac{1}{2} K_n' \frac{W_n}{L_n} (V_{GT})^2 = \frac{1}{2} K_p' \frac{W}{L} (V_{GT})^2$$

$$\frac{1}{2}(1.8\times10^{-4})\frac{0.45}{0.18}(V_{in}-0.5)^2 = \frac{1}{2}\, 3.6\times10^{-5}\left(\frac{1}{0.18}\right)(1.8-V_{in}-0.5)^2$$

$$450\,(V_{in}^2 - V_{in} + 0.25) = 200\,(V_{in}^2 - 2.6V_{in} + 1.69)$$

⇒ Solve quadratic for Vin

⇒ $V_{in} = 0.82V = V_M$

nmos: $V_{min} = 0.82 - 0.5 = 0.32$, so it is in normal saturation ✓

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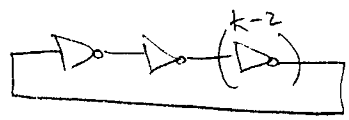
Schwindt Key



Problem 2 data

$\tau_{P_{L\to H}} = 29.77\,pS$

$\tau_{P_{H\to L}} = 26.23\,pS$

Assume 50% in gates Hi other Low thus this approx is valid

$$\tau_p = \frac{\tau_{P_{L\to H}} + \tau_{P_{H\to L}}}{2}$$

$\text{Period} = \tau_p \cdot k \cdot 2$ ← you must wait for twice the number of stages to have the oscillator "settle" to the final period

$\text{Period} = (55.5\,pS) \cdot k$

$$f_o = \frac{1}{\text{Period}} \Rightarrow \boxed{f_o = \frac{1.8018 \times 10^{10}}{k}\ Hz}$$

For one inverter

$P = I_{avg} V$

$$I_{avg} = \frac{I_{avg\,L\to H} + I_{avg\,H\to L}}{2} = \frac{168.83\,\mu A + 192\,\mu A}{2}$$

$I_{avg} = 180.415\,\mu A$

$P = (180.415\,\mu A)(1.8V)$

$P = 324.7\,\mu W$

$$\text{Power dissipation} = P \cdot k \Rightarrow \boxed{P_{diss} = (324.7\,\mu W) \cdot k}$$

With too few inverters in the oscillator the circuit may not achieve full voltage swings and eventually stabilize to a non-oscillatory state. One inverter will stabilize in the metastable state of Vdd/2 constantly dissipating power.